Graphing Quadratic Functions

About the Lesson

In this activity, students plot data and use the Transformation Graphing app to explore the parent function $y = x^2$. As a result, students will:

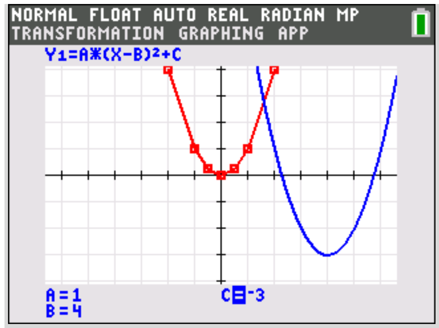
- Graph a quadratic function $y = ax^2 + bx + c$ and display a table for integer values of the variable.
- Graph the equation $y = a(x - h)^2$ for various values of a and describe its relationship to the graph of $y = (x - h)^2$.
- Determine the vertex, zeros, and the equation of the axis of symmetry of the graph $y = x^2 + k$ and deduce the vertex, the zeros, and the equation of the axis of symmetry of the graph of $y = a(x - h)^2 + k$

Vocabulary

- vertical dilation
- reflection
- vertical stretch
- vertical shrink

Teacher Preparation and Notes

- In this activity, students graph quadratic functions and study how the constants in the equations compare to the coordinates of the vertices and the axes of symmetry in the graphs. The first part of the activity focuses on the vertex form, while the second part focuses on the standard form.
- This activity uses the Transformation Graphing Application. Make sure that each calculator is loaded with this application before beginning the activity.
- Problem 1 introduces students to the vertex form of a quadratic equation, while Problem 2 introduces the standard form. You can modify the activity by working through only one of the problems.
- If you do not have a full hour to devote to the activity, work through Problem 1 on one day and then Problem 2 on the following day.
- Before beginning this activity, clear out any functions from the **Y=** screen and turn all plots off.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-84 Plus family with the latest TI-84 Plus operating system (2.55MP) featuring MathPrint ™ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

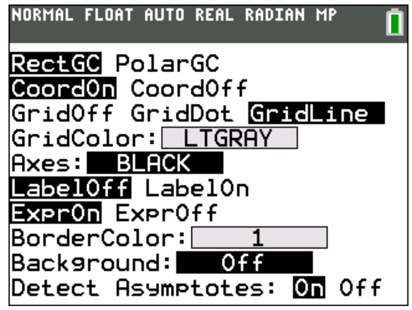
Associated Materials:

- GraphingQuadraticFunctions_Student.pdf
- GraphingQuadraticFunctions_Student.doc

Tech Tip: If your students are using the TI-84 Plus C Silver edition have them turn on the GridLine by pressing [2nd] [ZOOM] to change the [FORMAT] settings. If your students are using TI-84 Plus, they could use GridDot.

Problem 1 – The Parabola

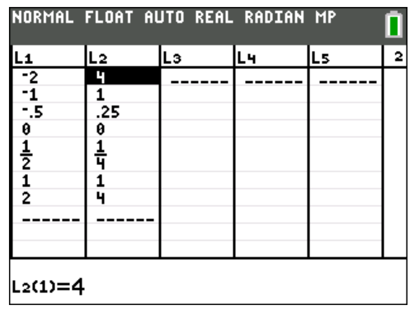
Students will examine the data in L_1 and L_2 shown on the student handout and to the right. Let L_1 be the *x*-value and L_2 be the *y*-values for a graph.



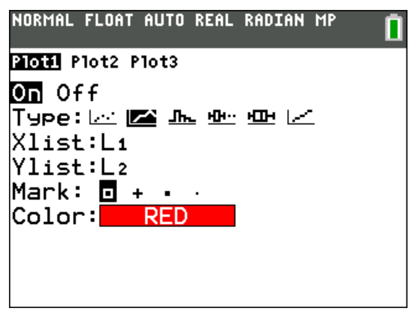
1. How are the *x* and *y*-values related? What pattern do you see?

Answer: The values in L_2 is the square of the corresponding values in the L_1 list.

To enter the data press [STAT] [ENTER]. In list L_1 enter –2, –1, –0.5, 0, ½, 1, 2. Press [ALPHA] [Y=] for [F1] to use the fraction template. Press the right arrow key so your cursor is on L_2 and type L_1^2. Press [2nd] [1] for [L1]. Now that the data is entered press [2nd] [Y=] for [STAT PLOT] and set up Plot1 as shown to the right. Press [ZOOM] and select **ZDecimal**. Additional suggestions and screenshots are given on the student activity sheet.



Now in [Y=] enter the parent function $y = x^2$ into $\mathbf{Y_1}$. Press [GRAPH]. This curve is called a parabola.



2. Describe the shape of the function.

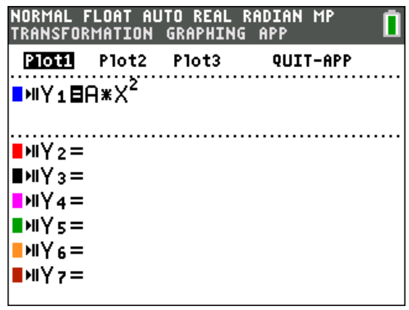
Answer: Students should recognize that the graph is not linear. It is curved upward. They may be reminded of the path of a projectile.

When students use the Transformation Graphing app only one function and up to three plots can be graphed at the same time. By plotting lists and the equation, students will be better able to see and compare the parent function during their exploration.

Teacher Tip: Be sure to clarify that changing the value of *a* in a function like $y = a \cdot f(x)$ causes a vertical dilation, which affects the *y*-values of the function.

Students may not know what a vertical dilation is. Multiplying a function by a constant causes a vertical dilation. It can be a vertical stretch or compression. A value of $a = -1$ is called a reflection. Be sure to encourage precision in the use of vocabulary.

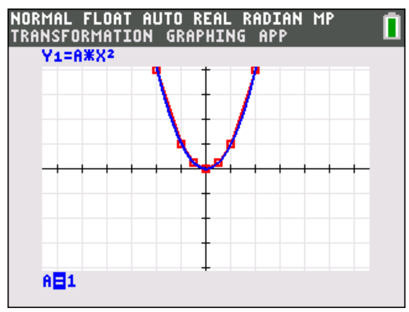
In geometry, a figure can be either enlarged or reduced using a scale factor while the center stays in the same position. Dilations of function graphs can be horizontal or vertical in nature. This document deals only with vertical dilations. Vertical dilations are commonly referred to as vertical stretch or vertical shrink. Horizontal dilations are of the form $y = f(b \cdot x)$. You may want to explore this more depending on the level of your students and the curriculum.



Now you will explore how values affect the characteristics of the parent function of the quadratic family of functions.

Press APPS. Choose Transfrm for the **Transformation Graphing** app. Press Y=, and ALPHA [A] to enter **A*X²** in **Y_1**. This is the equation of a parabola in vertex form.

Press GRAPH. Use the left and right arrows to explore different value of A.



3. Use the arrows to explore different values of A.

a. What is the value of A that makes the equation the parent function?

<u>**Answer:**</u> A = 1. The parent function is $y = x^2$. Therefore, $y = \text{A}\,x^2$ is equivalent to $y = x^2$ when so A = 1.

b. Describe what happens to the graph when A is greater than 1.

<u>**Answer:**</u> In the viewing window the graph appears steeper. Be sure to correct students if they claim the graph become narrower than the parent function. The correct term for this vertical dilation is a vertical stretch. See the teacher note above.

c. What happens when A is a negative number? Use the function to explain why this occurs.

<u>**Answer:**</u> Students should observe the graph turn down. The negative value for A makes the function negative. As the values for x get further from the origin, the y-values become more negative. When A = 1, the parent function is said to have reflected.

d. What happens to the graph when A = 0? Explain.

<u>**Answer:**</u> When A = 0, the graph is $y = 0x^2$. The graph y = 0 is a horizontal line.

e. Type in various values for A that are between 0 and 1, like ¼ and ½. Describe the shape.

<u>**Answer:**</u> This shape is called a vertical compression of the parent function.

4. Add another letter to explore. In $\mathbf{Y_1}$ enter $\mathbf{A(x - B)^2}$. Press GRAPH. Make A = 1 and again use the arrows or enter in a value for B to identify the pattern.

a. What happens when B is positive? When B is negative?

Answer: When B is positive, the lowest point of the graph is to the right of the *y*-axis. When B is negative, the lowest point of the graph is to the left of the *y*-axis.

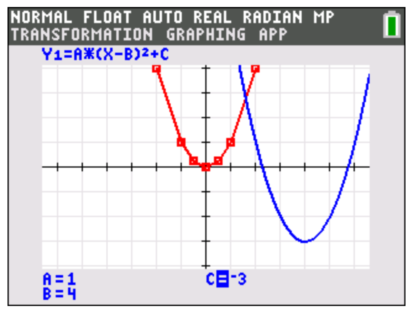
b. Describe the changes in the graph as B increases. What happens when B decreases?

Answer: As B increases the graph moves to the right. When B decreases the graph moves to the left. When the absolute value of *h* gets larger, the graph moves away from the *y*-axis. When the absolute value of *h* gets smaller, the graph moves closer to the *y*-axis.

Teacher Tip: You could also discuss what happens when the absolute value of B (or *h* in the typical general equation for a quadratic function in vertex form) increase. When the absolute value of *h* gets larger, the graph moves away from the *y*-axis. When the absolute value of *h* gets smaller, the graph moves closer to the *y*-axis.

5. In $\mathbf{Y_1}$ enter $\mathbf{A(x - B)^2 + C}$. Press WINDOW ▲ to display the **Settings Screen** and assign **A** = 1 and **B** = 0. Press GRAPH. Press ▼ to highlight **C**. What happens as you press ▶ to increase the value of **C**?

Answer: As C increases the graph goes up. This is a vertical shift.



6. Change B and C so that the vertex of the parabola is at (4, – 3).

a. Write your equation.

Answer: $y = (x - 4)^2 - 3$

b. The formula of a parabola can be written as $\boldsymbol{y = a(x - h)^2 + k}$. Why is this called *vertex form*? What affect does *a*, *h*, and *k* have on the graph? Relate the parameters *a*, *h*, and *k* to the parameters you explored.

Answer: The value of (*h*, *k*) is the vertex of the parabola. The lowercase *a* corresponds to A and causes the vertical dilation of stretching or compressing. The *h* = B in the transformation exploration. The *h* is for the horizontal shift. The constant *k* corresponded to C, the vertical shift.

Turn off the **Transformation Graphing** app ([APPS] **> Transfrm > Uninstall**).

7. Before graphing it, describe $y = ¼\,(x – 0)^2 – 3$. Then confirm your prediction by entering it in $\mathbf{Y_1}$.

Answer: Compared to the parent function, this graph will be shifted down 3 and has a vertical compression of ¼. The parabola opens up and has the vertex of (0, –3)

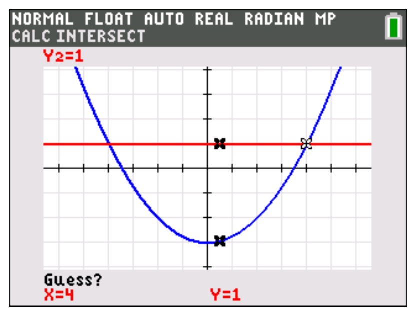
From [Y=] arrow up and press [ENTER] on Plot1 to turn it off.

8. Draw a line parallel to the *x*-axis that intersects the parabola in question 7 twice. Experiment with different equations in **Y2** until you find such a line. Record the equation in the first row of the table.

Sample Answers:

Line	Left intersection	Distance from left intersection to *y*-axis	Right intersection	Distance from right intersection to *y*-axis
y = 1	(–3 , 1)	4	(–4 , 1)	4
y = –2	(–2 , –2)	2	(2 , –2)	2
y = 3	(–5 , 3)	5	(–5 , 3)	5

If necessary, use the **intersect** command ([2nd] [CALC]) to find the coordinates of the two points where the line intersects the parabola. Record them in the table.



Choose a new line parallel to the *x*-axis and find the coordinates of its intersection with the parabola. Repeat several times, recording the results in the table.

9. Examine the table and make observations.

a. What do you notice about the points in the table? How do their *x*-coordinates compare? How do their *y*-coordinates compare?

Answer: The *x*-coordinates of the points are opposites of each other. The *y*-coordinates of the points are the same.

b. Calculate the distance from each intersection point to the *y*-axis. What do you notices about the distances from each intersection point to the *y*-axis?

Answer: The left and right points are equidistant from the *y*-axis.

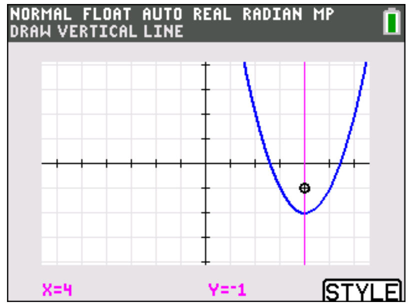
c. The relationships you see exist because the graph is symmetric and the *y*-axis is the *axis of symmetry*. What is the equation of the axis of symmetry?

Answer: $x = 0$

10. How do you think the axis of symmetry will change if h is changed from 0 to 4? Change the value of h in the equation in **Y1** from 0 to 4 Graph $y = (x - 4)^2 - 3$.

As before, enter an equation in Y2 to draw a line parallel to the x-axis that passes through the parabola twice. Find the two intersection points.

Answer: The answers will vary based on what the student graphs for the horizontal line. However, the two x-coordinates of the points should be equidistant from 4.



The axis of symmetry runs through the midpoint of these two points. Find the coordinates of the midpoint. (The midpoint occurs at $x = 4$.)

To draw this vertical line press [2nd] [DRAW] while on the graph and choose the **Vertical** command. This vertical line is the axis of symmetry.

11. a. What is the equation for the axis of symmetry?

Answer: $x = 4$ is the vertical line.

b. From the general equation $y = a(x - h)^2 + k$, what is the general equation of the axis of symmetry?

Answer: $x = h$ is the general equation of the axis of symmetry.

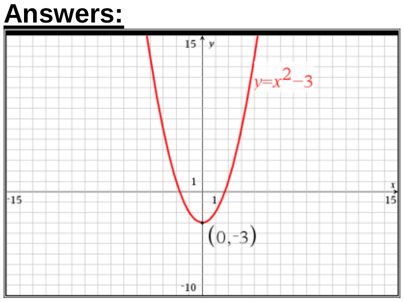
Sketch the graph of each function. Then check your graphs with your calculator. (You may need to adjust your viewing window.)

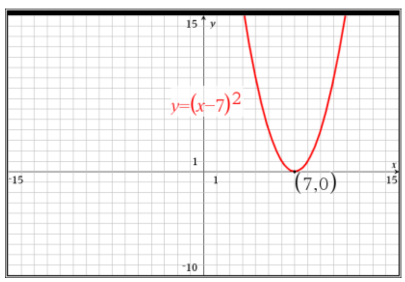
12. $y = x^2 - 3$

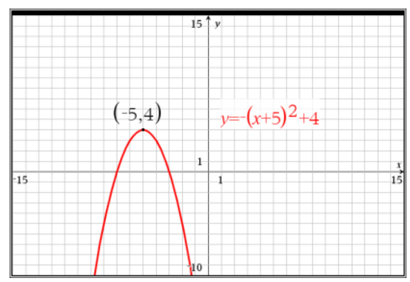
13. $y = (x - 7)^2$

14. $y = -(x + 5)^2 + 4$

Answers:

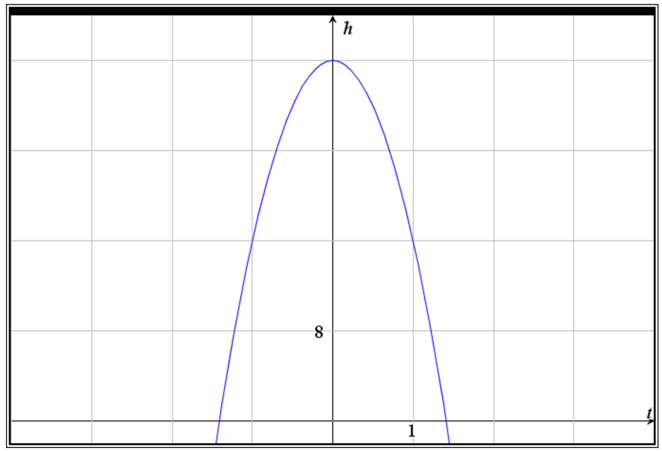






15. Galileo drops a cannonball out of a tower window 32 feet off the ground. The height *h* (in feet) of the cannonball at time *t* (in seconds) is given by $h = -16t^2 + 32$. Graph the parabola in the grid below. Note that the vertical tick marks are every 8 and the horizontal tick marks are 1. From your graph, estimate how long will it take for the cannonball to hit the ground. Confirm using algebra.

Answer: The vertex at (0, 32) and the graph open down. The zero is at about 1.4 sec. This works out algebraically as as $t = \pm\sqrt{2}$.



Problem 2 – Standard form

The standard form of a parabola is $y = ax^2 + bx + c$. Let's see how the standard form relates to the vertex form.

$$y = a(x-h)^2 + k$$
$$y = a(x^2 - 2xh + h^2) + k$$
$$y = \boxed{a}x^2 \boxed{-2ah}x + \boxed{ah^2 + k}$$

$$y = \boxed{a}x^2 + \boxed{b}x + \boxed{c}$$

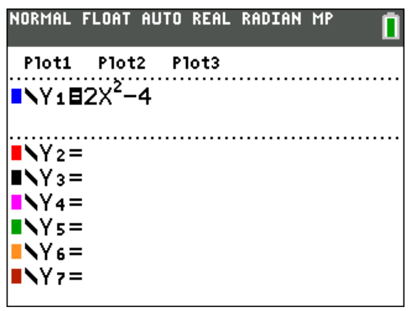
Using algebra we can see that the b in the general equation for the parabola written in standard form is related to the h in the following way.

$$b = -2ah$$
$$h = -\frac{b}{2a}$$

16. For the standard form of a parabola $y = ax^2 + bx + c$, what is the x-coordinate of the vertex? Give your answer in terms of the constants from the standard form.

Answer: $-\dfrac{b}{2a}$

17. The equation $y = 2x^2 - 4$ is in standard form. Graph this equation in $\mathbf{Y_1}$. Press ZOOM, select ZDecimal.



a. What is the value of a? Of b? Of c?

Answer: $a = 2$, $b = 0$, $c = -4$

b. What is the x-coordinate of the vertex?

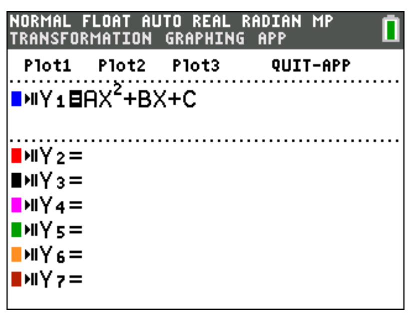
Answer: 0

18. Use the **minimum** command to find the vertex of the parabola.

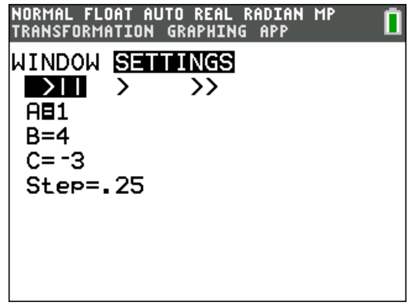
Answer: vertex: (0,4)

How do you think changing the coefficient of x^2 might affect the parabola?

Turn on the **Transformation Graphing** app and enter the equation for the standard form of a parabola in **Y1**.



Try different values of A in the equation. Make sure to test values of A that are between –1 and 1.



You can also adjust the size of the increase and decrease when you use the right and left arrows. Press [WINDOW] and arrow over to **Settings**. Then change the value of the step to 0.1 or another value less than 1.

19. Does the value of a change the position of the vertex?

Answer: No

20. How does the value of a relate to the shape of the parabola?

Answer: When a is positive, the parabola opens up. When a is negative the parabola opens down. The graph of $y = ax^2$ is a vertical stretch of the graph of $y = x^2$ by a factor of a for $a > 1$, and a vertical compression of the graph of the graph of $y = x^2$ by a factor of a for $a < 1$.

Find the y-intercept of the parabola. Use the **value** command ([2nd] [CALC]) to find the value of the equation at $x = 0$. Change the values of a, b, and/or c and find the y-intercept. Repeat several times and record the results in the table below.

Sample answers:

Equation	A	B	C	y-intercept
$y = 2x^2 - 4$	2	0	–4	–4
$y = -1x^2 + 2x + 3$	–1	2	3	3
$y = x^2 + x - 2$	1	1	–2	–2
$y = ½\,x^2 - 5x - 3$	½	–5	–3	–3

21. How does the equation of the parabola in standard form relate to the y-intercept of the parabola?

Answer: c is the y-intercept of the parabola.

Sketch the graph of each function. Then check your graphs with your calculator. (Turn off **Transformation Graphing** first. You may need to adjust your viewing window.)

22. $y = x^2 + 6x + 2$

23. $y = -x^2 - 4x$

24. $y = -2x^2 + 8x + 5$

Answer: