



## Lesson Overview

In this TI-Nspire lesson, students investigate how patterns in the computations for  $a^n$ ,  $(ab)^n$ ,  $(a^m)^n$  that held for positive integer exponents extend naturally to integer exponents.



Patterns in the computations for  $a^n$ ,  $(ab)^n$ ,  $(a^m)^n$  that held for positive integer exponents extend naturally to integer exponents.

## Learning Goals

1. Recognize properties of positive integer exponents extend to integer exponents;
2. apply properties of exponents to express numbers in scientific notation;
3. interpret and explain numerical expressions involving exponents that are 0, negative, or  $\frac{1}{2}$  and  $\frac{1}{3}$ .

## Prerequisite Knowledge

*Extending Exponents* is the eleventh lesson in a series of lessons that explores the concepts of expressions and equations. Students already should be familiar with the properties of positive integer exponents as developed through the lesson *What is an Exponent?* including the results of products raised to an exponent, and of numbers raised to successive exponents. Familiarity with decimal notation and place value are necessary for understanding scientific notation.

Knowledge of the vocabulary of squaring and cubing is needed to understand the notions of the square root or cube root of a number.

## Vocabulary

- **exponent:** a small number written above and to the right of the base number, telling how many times the base number is used as a factor.
- **base:** the number being multiplied.
- **factor:** numbers that are multiplied together to produce another number.
- **cube:** to raise a number to the third power.
- **square:** to raise a number to the second power.
- **power:** an expression that represents the repeated multiplication of the same factor.

## ⌚ Lesson Pacing

This lesson should span several class periods. The different parts of the lesson correspond to possible transition points between class periods.



## Lesson Materials

- Compatible TI Technologies:



TI-Nspire CX Handhelds,



TI-Nspire Apps for iPad®,



TI-Nspire Software

- Extending Exponents\_Student.pdf
- Extending Exponents\_Student.doc
- Extending Exponents.tns
- Extending Exponents \_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to <http://education.ti.com/go/buildingconcepts>.

## Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:



**Class Discussion:** Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.



**Student Activity:** Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.



**Deeper Dive:** These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

## Mathematical Background

Exponents play a major role in mathematics as building blocks for understanding and performing the operation of multiplication in purely numerical situations and in algebraic situations involving variables. In Grade 4, students gain familiarity with factors and multiples. In Grade 5, they use whole number exponents as a compact way to express powers of 10. In Grade 6, students expand these notions to use exponents as a compact way to write expressions involving whole number multiplication. The lesson *What is an Exponent?* focused on how the definition can be used in a variety of situations to make sense of the computations  $a^n$ ,  $(ab)^n$ ,  $(a^m)^n$  and where the exponents are positive integers, and in particular, allowed students to investigate whether exponents “distribute” over operations other than multiplication. *Extending Exponents* continues that development by considering integer exponents, including zero and negative integer powers, applied to rational number bases.



By convention, for any non-zero number  $a$ ,  $a^0 = 1$  and  $a^{-n} = \frac{1}{a^n}$ . Students investigate how patterns in the computations for  $a^n$ ,  $(ab)^n$ ,  $(a^m)^n$  that held for positive integer exponents extend naturally to integer exponents. Students are introduced to the particularly important application of integer exponents as used for scientific notation.

The *CCSS-M Progressions* document for Expressions and Equations refers to the “any order, any grouping” property, which in this case is a combination of the commutative and associative properties, indicating that any sequence of multiplications may be calculated in any order and that the numbers may be grouped together any way. Using this notion of “any order, any grouping” for multiplication along with “When in doubt, write it out”, students explore equivalent ways of writing numerical expressions involving exponents. In particular, rational numbers raised to an exponent always can be written in simplified form before writing as a product of repeated factors. (Note: the word *power* is sometimes used both as a synonym for *exponent* as well as the result of raising a number to an exponent. In this lesson, the word *power* is not used, to avoid unnecessary confusion. Another possible source of confusion is to write the reciprocal of a number  $a$  as  $\frac{1}{a}$ , especially when  $a$  itself is a fraction.)

A solid foundation in exponents is important because of the connection to exponential growth and decay, exponential functions in general, and to logarithms as the inverse of exponential functions. Common misconceptions include not understanding which element in the expression is the base of the exponent, multiplying the exponent and the base, and reversing the role of the base and the exponent.

This lesson provides a first introduction to work with non-integer rational exponents ( $\frac{1}{2}$  and  $\frac{1}{3}$ ) as alternative notations for square roots and cube roots, respectively. If a number can be written as the product of bases raised to even exponents, then that number raised to the  $\frac{1}{2}$  will result in the product of those same bases raised to exactly half of the original exponents. Similarly, if a number can be written as the product of bases raised to exponents that are multiples of 3, then that number raised to the  $\frac{1}{3}$  will result in the product of those same bases raised to exactly one-third of the original exponents.

Note that this lesson involves at least three major topics: extending exponents to integers, scientific notation, and square and cube roots. As a consequence, the lesson should span several class periods. The different parts of the lesson correspond to possible transition points between class periods.



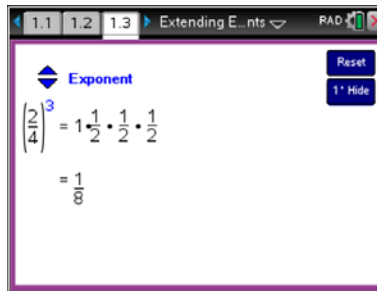
### Part 1, Page 1.3

Focus: Students explore the convention that, for any non-zero number  $a$ ,  $a^0 = 1$  and

$$a^{-n} = \frac{1}{a^n}.$$

On page 1.3, students can use the arrows on the screen or on the keypad to change the exponent.

Students can select the numerator or denominator of the base to erase the number and then type in a new number using the keypad.



### TI-Nspire Technology Tips

**menu** accesses page options.

**tab** toggles between the numerator and the denominator.

**ctrl del** resets the page.

**menu > Base** allows the base to be changed by typing a new base value.

**Show Keys** displays a small numerical keypad on screen.

**Hide Keys** hides the keypad.

**Reset** resets the page.



### Class Discussion

Have students...

**Look at page 1.3. Make a conjecture about each of the following, then check your conjecture using the TNS activity.**

- **What is the value of  $\left(\frac{2}{4}\right)^5$  ?  $\left(\frac{2}{4}\right)^1$  ?**

- **Change the base to  $\left(\frac{2}{1}\right)^5$ . Margo says that the value of this expression doubles every time you increase the exponent by 1, and the value is multiplied by  $\frac{1}{2}$  every time you decrease the exponent by 1. Is Margo right?**

Look for/Listen for...

Answer:  $\frac{1}{32}$  or  $\frac{32}{1024}$  ;  $\frac{1}{2}$  or  $\frac{2}{4}$ . Note: point out to students that writing  $\frac{2}{4}$  in simplified form,  $\frac{1}{2}$ , first results in a much easier computation than the “unsimplified” form, especially for larger exponents.

Answer: Margo is correct, because each time you increase the exponent, you multiply by another factor of  $\frac{2}{1}$ , and each time you decrease the exponent, a factor of  $\frac{2}{1}$  disappears, which gives the same result as multiplying by  $\frac{1}{2}$ .



## Class Discussion (continued)

- *Margo wonders what would happen if you decreased the exponent all the way down to 0. Based on her observation in the question above, what should the result be? Check your conjecture with the TNS activity.*

Answer: Since  $\left(\frac{2}{1}\right)^1 = 2$ , the value of  $\left(\frac{2}{1}\right)^0$  should be  $2\left(\frac{1}{2}\right) = 1$ .

- *What do you think the value of  $\left(\frac{2}{1}\right)^{-1}$  is? How about the value of  $\left(\frac{2}{1}\right)^{-5}$ ? Use Margo's observation to justify your reasoning. Check your answer with the TNS activity.*

Answer:  $\frac{1}{2}$  and  $\frac{1}{32}$ . The pattern Margo noticed continues: each time the exponent is decreased by 1, the result is multiplied by  $\frac{1}{2}$ .

Change the base to  $\frac{3}{2}$  and the exponent to 3.

- *Explain how the result changes as the exponent increases to 4, then to 5, then to 6.*
- *Now decrease the exponent back to 1. Explain how the result changes as you are decreasing the exponent by 1 each time.*
- *Predict what the result will be if the exponent is decreased by 1 more to reach 0.*

Answer: the result is multiplied by a factor of  $\frac{3}{2}$  each time the exponent increases by 1.

Answer: the result decreases by a factor of  $\frac{2}{3}$  each time, since a factor of  $\frac{3}{2}$  was removed with each decrease of 1 in the exponent.

Answer: When we reach an exponent of 0, only the factor 1 remains. Remember that the *reciprocal*, or *multiplicative inverse*, of a non-zero number  $a$  is  $\frac{1}{a}$ . The product of any non-zero number  $a$  and its reciprocal is 1.

Keep the base value of  $\frac{3}{2}$  and decrease the exponent to  $-1$ .

- *Joseph says that the result of  $\left(\frac{3}{2}\right)^{-1}$  is the reciprocal of  $\frac{3}{2}$ . Do you agree or with Joseph? Explain why or why not.*

Answer: Joseph is correct, because  $\left(\frac{3}{2}\right)^{-1} = \frac{2}{3}$  and  $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$ .



## Class Discussion (continued)

- **What do you think the value of  $\left(\frac{3}{2}\right)^{-2}$  is?**

Answer:  $\left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ .

**Check your answer with the TNS activity.**

- **What is the reciprocal of  $\left(\frac{3}{2}\right)^2$ ?**

Answer:  $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ , so its reciprocal is  $\frac{4}{9}$ , which is  $\left(\frac{3}{2}\right)^{-2}$ .

- **Joseph also says that for any number  $a$ , you could write its reciprocal as  $a^{-1}$ . Do you agree?**

Answer: Yes, as long as  $a$  is NOT zero. If  $a$  is non-zero, then  $a^{-1} = \frac{1}{a}$ , so it is another way of writing the reciprocal of  $a$ .

- **Use the TNS activity to decide which of the following is true. Explain why in each case.**

a.  $\left(\frac{3}{1}\right)^{-3} = -9$

b.  $\left(\frac{-3}{1}\right)^{-3} = -\frac{1}{27}$

c.  $(1000)^0 = 1$

d.  $(0)^{-1} = 0$

Answer: a) is not true because multiplying  $\frac{1}{3}$  as a factor 3 times gives you  $\frac{1}{27}$ . (Some students may mistakenly think that the result should be the product of 3 and -3, which is  $-9$ .)

b) is true because  $-\frac{1}{3}$  is a factor 3 times and  $-\frac{1}{27}$  is the result of multiplying those three factors together.

c) is true because any non-zero number to the zero power is 1.

d) is not true because  $(0)^{-1}$  represents  $\frac{1}{0}$  and division by 0 is undefined.

- **If  $a$  is a non-zero number, how would you explain the meaning of  $a$  raised to any integer power (positive, negative, or zero)?**

Answer: If  $n$  is a positive integer, then  $a^n = (a)(a)(a)\dots(a)$  where  $a$  appears as a factor  $n$  times;  $a^{-n} = \left(\frac{1}{a}\right)\left(\frac{1}{a}\right)\left(\frac{1}{a}\right)\dots\left(\frac{1}{a}\right)$  where the reciprocal  $\frac{1}{a}$  appears as a factor  $n$  times; and  $a^0 = 1$ .



## Student Activity Questions—Activity 1

1. Find at least three ways to write each of the following:

a.  $\frac{1}{64}$

Possible answers:  $\left(\frac{2}{1}\right)^{-6}$ ,  $\left(\frac{4}{1}\right)^{-3}$ ,  $\left(\frac{8}{1}\right)^{-2}$ ,  $\left(\frac{1}{2}\right)^6$ ,  $\left(\frac{1}{4}\right)^3$ ,  $\left(\frac{1}{8}\right)^2$

There are others!

b. 0.001

Possible answers:  $10^{-3}$ ,  $\left(\frac{1}{10}\right)^3$ ,  $\frac{1}{10^3}$ ,  $\frac{1}{1000}$ ,  $(0.1)^3$

## Part 2, Page 1.5

Focus: Students will explore ways to simplify expressions using the properties of exponents.

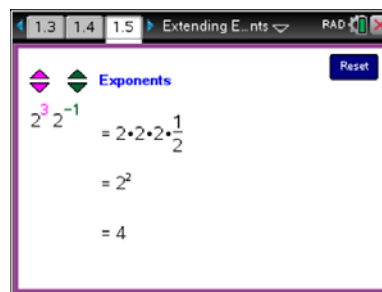
On page 1.5, use the arrows on the screen or on the handheld keypad to change the exponents.

**menu > Base** chooses which base will be changed by typing a new base value.

**Reset** resets the page.

**Show Keys** displays a small numerical keypad on screen.

**Hide Keys** hides the keypad.



## Class Discussion

*In your earlier work on exponents, you used “when in doubt, write it out”.*

- *How is that idea shown on page 1.5?*

Answer: The two factors  $2^3$  and  $2^{-1}$  are both written out using the definitions of positive and negative integer exponents:  $2^3 = 2 \cdot 2 \cdot 2$  and  $2^{-1} = \frac{1}{2}$ , the reciprocal of 2. When multiplied

together, one factor of 2 and the factor  $\frac{1}{2}$  multiply together to be 1, leaving  $2 \cdot 2 = 2^2 = 4$  as the final result.

- *Jayde got  $\frac{8}{2}$  as a final result. Is he correct?*

Answer: Yes, but Jayde could have simplified his answer to  $\frac{4}{1}$ , which matches the final result shown on the page.



## Class Discussion (continued)

- Find two different ways of changing the exponents to get a product equal to  $\frac{1}{64}$ .

Then for a product equal to  $2^{-3}$ .

Think about what positive and negative exponents mean and use this to write these products using a single exponent for a base of 2. (You can use the TNS activity to check parts a through c.)

- a.  $2^{-1}2^{-3}$  b.  $2^{-6}2^6$  c.  $2^{-4}2^{-6}$  d.  $2^{-150}2^{100}$  e.  $2^{253}2^{-253}$

Explain how  $2^3 2^{-4}$  can be written using only positive exponents as  $\frac{2^3}{3^4}$ . Then write each of the

following using only positive exponents in two different ways.

- a.  $4^3 \cdot 10^{-4}$       b.  $2^3 \cdot 4^{-4}$       c.  $2^3 \cdot 6^{-4}$

- d. Find a product of powers with two different bases that has a value that reduces to exactly 2.

Indicate whether each of the following is true or false.

- $5^2 2^{-3} = \frac{25}{8}$
- $5^{-6} 2^6 = 10^0 = 1$

Answer: There are many possible answers.

For example,  $\frac{1}{64} = 2^{-3} \cdot 2^{-3} = 2^{-4} \cdot 2^{-2} = 2^{-6} \cdot 2^0$

and  $2^{-3} = 2^2 \cdot 2^{-5} = 2^0 \cdot 2^{-3} = 2^{-1} \cdot 2^{-2}$ .

Answer: a.  $2^{-2}$ ; b.  $2^0 (=1)$ ; c.  $2^{-10}$ ; d.  $2^{-50}$ ; e.  $2^0 (=1)$

Answers:  $2^3 \cdot 3^{-4}$  is equal to  $2^3 \cdot \left(\frac{1}{3}\right)^4$  using the meaning of negative exponents, and this is the same as  $\frac{2^3}{3^4}$ .

$$a. \frac{4^3}{10^4} = \frac{4 \cdot 4 \cdot 4}{10 \cdot 10 \cdot 10 \cdot 10} = \frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5 \cdot 10} = \frac{2^3}{5^3 \cdot 10^1}$$

$$b. \frac{2^3}{4^4} = \frac{2 \cdot 2 \cdot 2}{4 \cdot 4 \cdot 4 \cdot 4} = \frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2 \cdot 4} = \frac{1}{2^3 \cdot 4^1} = \frac{1}{2^5}$$

$$c. \frac{2^3}{6^4} = \frac{2 \cdot 2 \cdot 2}{6 \cdot 6 \cdot 6 \cdot 6} = \frac{1 \cdot 1 \cdot 1}{3 \cdot 3 \cdot 3 \cdot 6} = \frac{1}{3^3 \cdot 6^1} = \frac{1}{3^4 \cdot 2^1}$$

d. Answers can vary. Two possibilities are  $6^1 3^{-1}$  and  $4^2 2^{-3}$ .

Answer: true

Answer: false;  $5^{-6} 2^6 = \frac{2^6}{5^6} = \frac{64}{15625}$

(Note: The statement represents a common error: combining both multiplying the bases (permissible with a common exponent) and adding exponents (permissible with a common base)).





### Class Discussion (continued)

- $2^{-4}4^{-2} = 8^8$

Answer: false;  $2^{-4}4^{-2} = \frac{1}{2^4} \cdot \frac{1}{4^2} = \frac{1}{16} \cdot \frac{1}{16} = \frac{1}{256}$

(Note: The statement represents another common error: multiplying both the bases and exponents.)

- $2^{-150}3^{100} = \left(\frac{3}{2}\right)^{100} \left(\frac{1}{2}\right)^{50}$

Answer: true;

$$2^{-150}3^{100} = \frac{3^{100}}{2^{150}} = \frac{3^{100}}{2^{100}} \cdot \frac{1}{2^{50}} = \left(\frac{3}{2}\right)^{100} \left(\frac{1}{2}\right)^{50}$$

- $2^{253}4^{-253} = 2^{-253}$

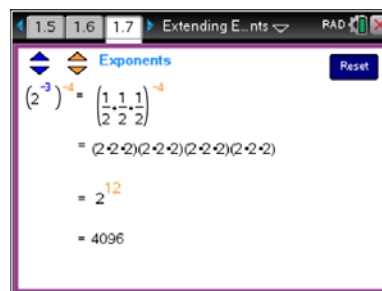
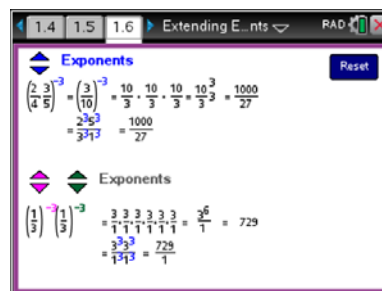
Answer: true;

$$2^{253} \cdot 4^{-253} = \frac{2^{253}}{4^{253}} = \left(\frac{2}{4}\right)^{253} = \left(\frac{1}{2}\right)^{253} = 2^{-253}$$

### Part 3, Pages 1.6 and 1.7

Focus: Students extend distribution over multiplication to negative exponents.

On pages 1.6 and 1.7, students can use the arrows on the screen or on the handheld keypad to change the exponents. Navigation functions similar to the navigation on page 1.5.



### Class Discussion

Have students...

Move to page 1.6.

- The top line equates the first expression**  
 $\left(\frac{2}{4} \cdot \frac{3}{5}\right)^{-3}$  to  $\left(\frac{3}{10}\right)^{-3}$ . **Where does  $\frac{3}{10}$  come from?**

Look for/Listen for...

Answer: The product inside the parentheses was computed first:  $\frac{2}{4} \cdot \frac{3}{5} = \frac{6}{20} = \frac{3}{10}$ .



## Class Discussion (continued)

- Explain why  $\left(\frac{3}{10}\right)^{-3} = \left(\frac{10}{3}\right)\left(\frac{10}{3}\right)\left(\frac{10}{3}\right)$

Answer: The negative exponent  $-3$  indicates that the reciprocal of  $\frac{3}{10}$ , namely  $\frac{10}{3}$ , should be raised to the exponent 3, or equivalently,  $\left(\frac{10}{3}\right)\left(\frac{10}{3}\right)\left(\frac{10}{3}\right)$ .

- The lower part of the page shows three factors of  $\frac{2}{1}$ . Where do these factors come from?

Answer: The first factor of the product is  $\left(\frac{2}{4}\right)^{-3}$ . The negative exponent,  $-3$ , indicates that the reciprocal of  $\frac{2}{4}$ , namely  $\frac{4}{2}$  or equivalently  $\frac{2}{1}$ , should be raised to the exponent 3, or equivalently,  $\left(\frac{2}{1}\right)\left(\frac{2}{1}\right)\left(\frac{2}{1}\right)$ .

- Explain why it is reasonable that the two expressions  $\left(\frac{2}{4} \cdot \frac{3}{5}\right)^{-3}$  and  $\left(\frac{2}{4}\right)^{-3} \cdot \left(\frac{3}{5}\right)^{-3}$  have the same value.

Answer: If the reciprocals of the product of the fractions in parentheses in the first line are written out as factors repeated three times, and the reciprocals of the same but separate fractions in the parentheses at the bottom also are written out as factors repeated three times each, then the order does not make any difference due to the commutative property of multiplication. The final resulting products will have the same value.

**Make a conjecture about what you think will happen in each case. Explain your reasoning. Then check your conjecture using the TNS activity.**

- All of the exponents in both expressions are changed to  $-5$ .

Answer: Two more factors of the reciprocal of product of  $\frac{2}{4}$  and  $\frac{3}{5}$ , namely  $\frac{10}{3}$ , will show up in the top expression and two more factors each of the separate reciprocals of  $\frac{2}{4}$  and  $\frac{3}{5}$  will show up in the bottom expression, resulting in the same final resulting product (of  $\frac{100000}{243}$ ). Some students might say that exponents still distribute over multiplication, even when they are negative.



## Class Discussion (continued)

- *The exponents for both bases in the bottom expression are changed to 5, but the exponent in the top expression is left as  $-5$ .*

Answer: The value of the bottom expression will now be  $\frac{243}{100000}$ , exactly the reciprocal of the value of the top expression. This makes sense, since every factor appearing in the top expression is the reciprocal of the product of a corresponding pair of factors in the bottom expression.

One also could argue that the bottom expression is equivalent to  $\left(\frac{2}{4} \cdot \frac{3}{5}\right)^5$ , so it must be the reciprocal of the top expression  $\left(\frac{2}{4} \cdot \frac{3}{5}\right)^{-5}$ .

- *Reset page 1.6. Change the first fraction from  $\frac{2}{4}$  to  $\frac{2}{7}$  in both the top and bottom expressions and change all the exponents to  $-1$  and press enter.*

*Bernardo says, "Look, multiplying two numbers and then finding the reciprocal gives the same answer as finding the two reciprocals first and then multiplying."*

*Do you agree or disagree?*

- *Chelsea then changed all the exponents to  $-2$  to see what would happen. Then she changed all the exponents to  $-3$ , and then all to  $-4$ . Chelsea says, "Negative exponents distribute over multiplication." Explain Chelsea's statement.*

Answer: Bernardo is correct, as long as neither of the numbers is 0. The reciprocal of a product of two non-zero numbers is the product of their reciprocals.

Answer: Chelsea is saying that a negative exponent applied to a product of two numbers multiplied together will be equal the final result obtained from applying the same negative exponent to the individual numbers first, and then multiplying those results together. (Chelsea's statement is related to Bernardo's, just with more factors involved due to the exponents.)

**Move to page 1.7.**

- *Where does the  $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$  inside the parentheses on the first line come from?*

Answer: The  $2^{-3}$  in parentheses of the original expression indicates that the reciprocal of 2, namely  $\frac{1}{2}$ , should be raised to the exponent 3, or  $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$ .



### Class Discussion (continued)

- **Explain where the second line on the page comes from.**

Answer: The exponent  $-4$  indicates that the reciprocal of  $(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2})$ , namely  $(2 \cdot 2 \cdot 2)$ , should be raised to the exponent 4, or  $(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$ .

- **Set both of the exponents (inside and outside the parentheses) on the original expression to  $-1$ .**

**Bernardo says, "This means the reciprocal of the reciprocal is just the number you started with."**

Answer: Yes, Bernardo is correct, as long as the original number is not 0. Bernardo is noting that  $2^{-1}$  is the reciprocal of 2, namely  $\frac{1}{2}$ , meaning that

$2 \cdot \frac{1}{2} = 1$ . But that also means 2 is the reciprocal of

$$\frac{1}{2}, \text{ so } (2^{-1})^{-1} = \left(\frac{1}{2}\right)^{-1} = \frac{2}{1} = 2.$$

- **Chelsea tries several other combinations of negative exponents and says, "The result is always the original number to a positive power."**

Answer: Chelsea is correct, since the two successive negative exponents always will result in repeated factors of "the reciprocal of the reciprocal," or simply the original number. Students should note that, using the basic definitions for negative and zero exponents and the "when in doubt, write it out" principle is all you really need to expand powers of products or successive powers of numbers. (The multiplication rules for positive integer exponents continue to work for negative integer and zero exponents.)

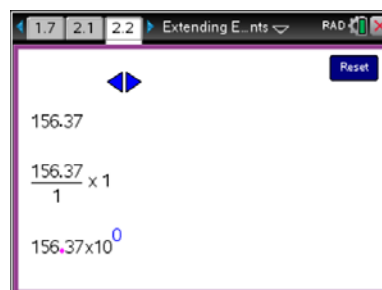
**Is he correct? Explain why or why not.**

**Is she correct? Explain why or why not.**

### Part 4, Pages 2.2 and 2.3

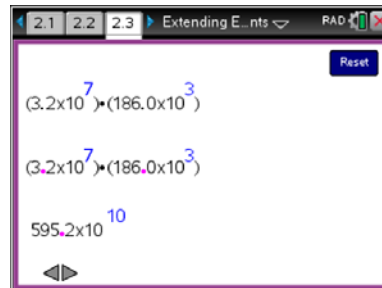
Focus: Scientific notation is a special application of integer exponents.

On page 2.2, students can use the arrows on the screen or on the keypad to change the denominator and multiplier simultaneously on the second line and to move the decimal point and change the power of 10 on the third line. Grab and drag the decimal point on the third line to change the power of 10 and the multiplier and denominator in the second line.



(Note: additional leading or zeroes can be added during editing to widen the range of place the decimal point can move.)

On page 2.3, students can use **tab** and the arrows on the handheld keypad to change the decimal point in the factors and the product. Select the factors in the product to edit. The pink decimal points can be dragged to change the factors and exponents.





## Class Discussion

Have students...

Open page 2.2.

- ***These three expressions all represent the same number. Explain why.***
- ***Select the left arrow once. What changes in the third line?***
- ***Explain how the changes in the second line are related to the changes in the third line.***
- ***Explain why these three expressions still represent the same number.***

Select the left arrow once more.

- ***What changes in the third line?***
- ***How do the changes in the third line relate to the changes in the second line?***

Look for/Listen for...

Answer: The second line is the same number as the first line, since multiplying or dividing any number by 1 will result in that same number. The third line just shows another way of writing the multiplication by 1, since  $10^0$  is 1.

Answer: The decimal point has moved to the left one place, and the exponent on 10 in the second factor has changed from 0 to 1.

Answer: The second line shows division by 10 followed by multiplication by 10. The decimal number on the third line is the result of the division by 10, and the multiplied 10 in the second line is the same as the  $10^1$  in the third line.

Answer: Dividing by 10 and then multiplying by 10 is the same as multiplying by  $\frac{10}{10}$ , which is just multiplying by 1, so the second line is the same number as the first line. The third line just shows this division and multiplication by 10 in a different way, so it is also the same number.

Answer: The decimal point in the first factor has moved one more place to the left and now the exponent on 10 is 2.

Answer: The second line shows division by 100 followed by multiplication by 100. The decimal number on the third line is the result of the division by 100, and the multiplied 100 in the second line is the same as the  $10^2$  in the third line.



## Class Discussion (continued)

- **Do the three expressions still represent the same number? Explain why or why not.**

Answer: Dividing by 100 and then multiplying by 100 is the same as multiplying by  $\frac{100}{100}$ , which is

just multiplying by 1, so the second line is the same number as on the first line. The third line just shows this division and multiplication by 100 in a different way, so it is also the same.

- **Predict what changes will result from selecting the left arrow once more. Check by using the TNS activity.**

Answer: The second line shows division and multiplication by 1000. The third line shows the decimal point in the first factor has moved one more place to the left and the other factor 1,000 is now written as  $10^3$ .

**Reset. Select the right arrow once.**

- **Now what changes in the second and third lines?**

Answer: The second line shows division and multiplication by 0.1, and in the third line the decimal point has moved one place to the right in the first factor and the exponent on 10 in the second factor has changed to  $-1$ .

- **$\frac{156.37}{0.1}$  appears as the first factor on the second line. What number could you multiply both the numerator and denominator by to see that this is equal to the first factor 1563.7 on the third line?**

Answer: If you multiply 156.37 and 0.1 both by 10, you would get an equivalent fraction of  $\frac{156.37}{0.1} = \frac{10(156.37)}{10(0.1)} = \frac{1563.7}{1}$ , which is the same as 1563.7 shown on the third line.

- **Explain why  $0.1$  and  $10^{-1}$  are equal. What does this mean about the numerical values shown on the three lines?**

Answer:  $10^{-1} = \frac{1}{10}$  using the definition of negative exponents, and 0.1 is the same as  $\frac{1}{10}$  in decimal form. This means the numerical values on all three lines are equal.

**Select the right arrow once again.**

- **Now what changes in the second and third lines?**

Answer: The second line shows division and multiplication by 0.01, and in the third line the decimal point has moved one more place to the right in the first factor and the exponent on 10 in the second factor has changed to  $-2$ .



## Class Discussion (continued)

- **Explain why the expressions on the second and third lines still equal the number on the first line.**

Answer: Division by 0.01 followed by multiplication by 0.01 will be the same as multiplying by 1, so the number in the second line equals the number in the first line.

The first factor on the third line is equivalent to the first factor on the second line, and the second factor on the third line is  $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$ , which is equal to the second factor of 0.01 on the second line.

- **The TNS activity is set up so that the decimal point does not move beyond the last digit showing at the right. Select the digits to the right of the decimal point on the number on the first line and type in 370 to get 156.370 and press enter. Then predict what will appear if you select the right arrow three times. Check your prediction with the TNS activity.**

Answer: The second line shows a division of the original number 156.370 by .001 followed by a multiplication by .001, while the third line shows the  $156370 \times 10^{-3}$ .

**A special notation used in many applications of science is scientific notation. It is especially handy for communicating very large or very small quantities.**

**In scientific notation, the number 156.37 would be written as  $156.37 \times 10^2$ , a product of two factors. The first factor must be a number greater than or equal to 1 but strictly less than 10, and the second factor is 10 to an integer exponent.**

**Open page 2.3. Two number factors are shown, each written as a decimal number times a power of 10.**

- **The product of the two factors is shown at the bottom of the page. How would you describe the way it is computed?**
- **Which of the three numbers (two factors and the product) is already written in scientific notation?**

Answer: The decimal number is the product of the two decimal numbers in the factors and the power of 10 is the product of the powers of 10 in the factors.

Answer: The first factor is already in scientific notation, since it is written as  $3.2 \times 10^7$ , a decimal number between 1 and 10 times an integer power of 10. The second factor and the product are not written in scientific notation.



## Class Discussion (continued)

- **Tab to the second factor and use the arrow keys to rewrite it in scientific notation on the second line. Press enter to recalculate the product. Is the product in scientific notation now?**
- **Monica says that if two numbers are written in scientific notation, then the product computed here always will be in scientific notation. Do you agree? If not, can you find an example that shows that Monica is not correct?**

Answer: The second factor in scientific notation is  $1.860 \times 10^5$ . The recalculated product is  $5.952 \times 10^{12}$  and is in scientific notation.

Answer: Sometimes, like the example here, the product would be in scientific notation. However, two decimal numbers between 1 and 10 could have a product bigger than 10.

For example,  $(7.0 \times 10^5)(6.0 \times 10^3) = 42 \times 10^8$ .

The two factors are in scientific notation, but the product is not in scientific notation. The product would need to be written as  $4.2 \times 10^9$  to be in scientific notation.



## Student Activity Questions—Activity 2

1. Write the following in scientific notation. Use the TNS activity to check your answers.

a. 345,000

b. 0.0007902

c. 3.14159

Answers: a.  $3.45 \times 10^5$ , b.  $7.902 \times 10^{-4}$ , c.  $3.14159 \times 10^0$

2. Each of the following problems involves very large or small numbers.

a. Human hair grows at a rate  $1.98 \times 10^{-4}$  meters/hour. How long does human hair grow in a year?

Answer: A year has 365 days and 24 hours in each day, so a year is  $365 \times 24 = 8760 = 8.76 \times 10^3$  hours long. Human hair will grow  $(8.76 \times 10^3)(1.98 \times 10^{-4}) = 17.3448 \times 10^{-1} \approx 1.7345 \times 10^0$ , or approximately 1.7345 meters in a year.

b. Juno sent a message from its orbit around Jupiter (540,000,000 miles away) in July 2016. Radio waves travel at the speed of light, which is 186,000 miles per second. How long did the message take to make the trip? Express your answer in seconds using scientific notation. How many minutes is this?

Answer:  $\frac{5.4 \times 10^8}{1.86 \times 10^5} = 2.90322 \times 10^3$  seconds. This is approximately 2,903 seconds and there are

60 seconds in a minute, so it takes approximately  $\frac{2903}{60} \approx 48.4$  minutes for the message to reach Earth.





## Student Activity Questions—Activity 2 (continued)

- c. One mole of hydrogen has a mass of 1.008 grams. If there are approximately  $6.022 \times 10^{23}$  hydrogen atoms in one mole, what is the approximate mass of one hydrogen atom?

Answer:  $\frac{1.008 \times 10^0}{6.022 \times 10^{23}} \approx 0.1674 \times 10^{-23} \approx 1.674 \times 10^{-24}$  grams.

- d. The United States' budget deficit in 1998 was  $\$5.653 \times 10^{12}$ . The budget deficit is projected to be 19.3 trillion dollars at the end of 2016. How many times larger will the budget deficit be then, compared to 1998?

Answer: \$19.3 trillion is  $\$19,300,000,000,000 = \$1.93 \times 10^{13}$ , so the deficit at the end of 2016 will be  $\frac{1.93 \times 10^{13}}{5.653 \times 10^{12}} \approx 0.3414 \times 10^1 \approx 3.414 \times 10^0$ , or roughly three and half times larger than the budget deficit in 1998.

### Part 5, Pages 3.2 and 3.3

Focus: Exponents  $\frac{1}{2}$  and  $\frac{1}{3}$ , square roots and cube roots

Navigation on pages 3.2 and 3.3 functions in the same way as the navigation on page 1.5.

**Exponent/Square Root** or **Exponent/Cube Root** toggles between a display of exponent notation ( $\frac{1}{2}$  or  $\frac{1}{3}$ ) and a display of the radical notation (square root or cube root symbol).

Calculator display for Square Root:  $36^{\frac{1}{2}} = (2 \cdot 2 \cdot 3 \cdot 3)^{\frac{1}{2}}$   
 $= ((2 \cdot 2) \cdot (3 \cdot 3))^{\frac{1}{2}}$   
 $= (2 \cdot 3) \cdot (2 \cdot 3)^{\frac{1}{2}}$   
 $= 2 \cdot 3$

Calculator display for Cube Root:  $216^{\frac{1}{3}} = (2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3)^{\frac{1}{3}}$   
 $= ((2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3))^{\frac{1}{3}}$   
 $= (2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3)^{\frac{1}{3}}$   
 $= 2 \cdot 3$



### Class Discussion

*To square a number means to raise it to the exponent 2. For example,  $6^2 = 36$ . If you write 6 as a product  $2 \cdot 3$ , then you could also write  $6^2 = (2 \cdot 3)^2 = (2 \cdot 3)(2 \cdot 3) = 36$ .*

*Notice that the factors of 6 (2 and 3) appear twice as many times in the  $6^2 = 36$ .*



## Class Discussion (continued)

- **Open page 3.2. To raise a number to the exponent  $\frac{1}{2}$  means to take each of its factors half as many times as it appears and multiply these together to get the result.**

**Use the page to explain why  $36^{\frac{1}{2}} = 6$ .**

- **Select the Exponent button to switch over to square root notation. The notation  $\sqrt{36}$  is simply another way of writing  $36^{\frac{1}{2}}$ . To find the square root of a number means to find another number whose square is the original number. Change the 36 to 50, Show Keys, and press enter.**

**Why does the page display “Value is not an integer” for  $\sqrt{50}$  ?**

- **Edit the base again to be 400 and Enter. Explain the result.**

Answer: There are several ways students might use the page to explain this.

1)  $36 = (2)(2)(3)(3)$ , so 36 has two factors of 2 and two factors of 3. This means that  $36^{\frac{1}{2}}$  should have just one factor of 2 and one factor of 3, giving us  $36^{\frac{1}{2}} = 2 \cdot 3 = 6$ .

2)  $36 = (2 \cdot 3)(2 \cdot 3) = (6)(6)$ , so 36 has two factors of 6. This means that  $36^{\frac{1}{2}}$  should have just one factor of  $2 \cdot 3$  or 6, and  $36^{\frac{1}{2}} = 2 \cdot 3 = 6$ .

Answer:  $50 = (2)(5)(5)$ . There are two factors of 5, but only one factor of 2, so it is not possible to take exactly half the number of each factor.

Answer:  $400 = (2)(2)(2)(2)(5)(5)$ , so it has 4 factors of 2 and 2 factors of 5. It is possible to take exactly half as many of each factor, namely 2 factors of 2 and 1 factor of 5 so that  $\sqrt{400} = 2 \cdot 2 \cdot 5 = 20$ .

**To cube a number means to raise it to the exponent 3. For example,**

**$6^3 = 6 \cdot 6 \cdot 6 = 216$ . If you write 6 as a product  $2 \cdot 3$ , then you could also write**

$$6^3 = (2 \cdot 3)^3 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = 216$$

**Notice that the factors of 6 (2 and 3) appear three times as often in  $6^3 = 216$ .**



## Class Discussion (continued)

- **Move to page 3.3. To raise a number to the exponent  $\frac{1}{3}$  means to take each of its factors one-third as many times as it appears and multiply these together to get the result.**

**Use the page to explain why  $216^{\frac{1}{3}} = 6$ .**

Answer: There are several ways students might use the page to explain this.

1)  $216 = (2)(2)(2)(3)(3)(3)$ , so 216 has three factors of 2 and three factors of 3. This means that  $216^{\frac{1}{3}}$  should have just one factor of 2 and one factor of 3, giving us  $216^{\frac{1}{3}} = 2 \cdot 3 = 6$ .

2)  $216 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = (6)(6)(6)$ , so 216 has three factors of 6. This means that  $216^{\frac{1}{3}}$  should have just one factor of  $2 \cdot 3$  or 6, and  $216^{\frac{1}{3}} = 2 \cdot 3 = 6$ .

- **Select the Exponent button to switch over to cube root notation. The notation  $\sqrt[3]{216}$  is simply another way of writing  $216^{\frac{1}{3}}$ . To find the cube root of a number means to find another number whose cube is the original number. Change the 216 to 81, Show Keys, and press enter. Why does the page display “Value is not an integer” for  $\sqrt[3]{81}$ ?**
- **Edit the base again to be 729 and press enter. Explain the result.**

Answer:  $81 = (3)(3)(3)(3)$ . There are four factors of 3, so it is not possible to take exactly one-third the number of factors.

Answer:  $729 = (3)(3)(3)(3)(3)(3)$ , so it has 6 factors of 3. Now it is possible to take exactly one-third as many factors of 3, namely 2 factors of 3 so that  $\sqrt[3]{729} = 3 \cdot 3 = 9$ .



### Deeper Dive — Page 1.3

**Barney says that  $0^0$  should be equal to 1, since any other number to the zero power is 1, but Juan says that  $0^0$  should be equal to 0, since 0 to any other power is 0. What do you think?**

Discussion: There are likely to be students on both sides, since both Barney's and Juan's arguments both sound reasonable. However, either of these choices would lead to results inconsistent with how exponents behave. For example, if we sided with Barney and agreed that  $0^0$  should be 1, then  $\frac{3^0}{0^0} = \frac{1}{1}$ , but on the other hand,  $\frac{3}{0}$  is undefined (division by 0 does not make sense), so  $\left(\frac{3}{0}\right)^0$  would not be defined either. (The discussion of why division by 0 is undefined may lead to reconsidering the relationship between the equations  $\frac{a}{b} = c$  and  $a = bc$ , and thinking about  $a = 3$ ,  $b = 0$ , and  $c$  being any number.) If we sided with Juan and agreed that  $0^0$  should be 0, then that result is inconsistent with  $a^0 = 1$  for every other value  $a \neq 0$ . (Note: The TNS activity also reports that  $0^0$  is undefined.)

### Deeper Dive — Page 1.5

**How many different ways could the exponents be changed to get a product equal to 1? Explain.**

Answer: There are infinitely many ways (though the TNS activity will only let you try exponents from  $-12$  to  $12$ ). If  $n$  is any integer, then  $2^n \cdot 2^{-n} = 2^0 = 1$ .

### Deeper Dive — Page 1.6

**Do negative exponents distribute over division, too? Explain why or why not.**

Discussion: As long as the division is not by 0, the answer is yes. This could be illustrated with the TNS activity, since division by any non-zero number  $a$  is equivalent to multiplication by the reciprocal  $\frac{1}{a}$ .

For example,  $\left(\frac{2}{7} \div \frac{5}{3}\right)^{-3} = \left(\frac{2}{7} \cdot \frac{3}{5}\right)^{-3}$  and

$\left(\frac{2}{7}\right)^{-3} \div \left(\frac{5}{3}\right)^{-3} = \left(\frac{2}{7}\right)^{-3} \cdot \left(\frac{5}{3}\right)^3 = \left(\frac{2}{7}\right)^{-3} \cdot \left(\frac{3}{5}\right)^{-3}$  and

we have seen that these give the same final result (by Chelsea's observation).



Deeper Dive — Page 3.2

**Jeremiah notes that  $\sqrt{49} = 7$  and  $\sqrt{64} = 8$ , so that while  $\sqrt{50}$  does not have an integer value, it must be some number that is between 7 and 8, and this number is probably just a little bit bigger than 7. What do you think about Jeremiah's claim?**

Discussion: When a number does not have an integer square root, it must lie between two consecutive “perfect squares,” which provides a means of estimating the square root to lie between two consecutive integers. Students might think that there is some rational number between 7 and 8 that is the square root of 50. However, for any positive integer, if it is not a perfect square (so that its square root is another positive integer), then its square root is irrational (meaning that it cannot be written as fraction with integer numerator and denominator).

Deeper Dive — Page 3.3

**Selma notes that this TNS activity does not allow one to enter a negative number for the base, but she claims that the cube root of  $-8$  should be  $-2$ , since  $(-2)(-2)(-2) = -8$ . Do you agree?**

Discussion: Selma is correct, and this brings up a key difference between square roots and cube roots. Since the square of any real number will be non-negative, the square root of a negative number will not exist (in the real numbers). But the product of three negative numbers is again negative, and the cube root of a negative number can exist, and will be negative. When a number does not have an integer cube root, it must lie between two consecutive “perfect cubes,” which provides a means similar to Jeremiah's for estimating the cube root to lie between two consecutive integers. For any integer, if it is not already a perfect cube (so that its cube root is another integer), then its cube root is irrational (meaning that it cannot be written as fraction with integer numerator and denominator).



## Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. The body of a 154-pound person contains approximately  $2 \times 10^{-1}$  milligrams of gold and  $6 \times 10^3$  milligrams of aluminum. Based on this information, the number of milligrams of aluminum in the body is how many times the number of milligrams of gold in the body?

**Answer:**  $3 \times 10^4$

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2. Approximately  $7.5 \times 10^5$  gallons of water flow over a waterfall each second. There are  $8.6 \times 10^4$  seconds in one day. Select the approximate number of gallons of water that flow over the waterfall in one day.

- a.  $6.45 \times 10^{21}$     b.  $6.45 \times 10^{20}$     c.  $6.45 \times 10^{10}$     d.  $6.45 \times 10^9$

**Answer:** c

3. Which of the following expressions are equivalent to  $\frac{3^{-8}}{3^{-4}}$ ? Select all that apply.

- a.  $3^{-12}$     b.  $3^{-4}$     c.  $3^2$     d.  $\frac{1}{3^2}$     e.  $\frac{1}{3^4}$     f.  $\frac{1}{3^{12}}$

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**Answer:** b and e

4. A carpenter bought 750 nails. Each nail has a mass of  $5.2 \times 10^{-3}$  kilogram. What is the total mass, in kilograms, of the nails the carpenter bought? Give your answer as a decimal.

**Answer:**  $3900 \times 10^{-3} = 3.9$

5. Order these numbers from least to greatest:

- a.  $7.50 \times 10^{-5}$     b.  $3.22 \times 10^{-7}$     c.  $8.00 \times 10^{-10}$     d.  $6.13 \times 10^{-5}$

**Answer:** c, b, d, a



## Student Activity Solutions

In these activities, you will use exponents as a compact way to write expressions involving whole number multiplication. After completing the activities, discuss and/or present your findings to the rest of the class.



### Activity 1 [Page 1.3]

1. Find at least three ways to write each of the following:

a.  $\frac{1}{64}$

Possible answers:  $\left(\frac{2}{1}\right)^{-6}$ ,  $\left(\frac{4}{1}\right)^{-3}$ ,  $\left(\frac{8}{1}\right)^{-2}$ ,  $\left(\frac{1}{2}\right)^6$ ,  $\left(\frac{1}{4}\right)^3$ ,  $\left(\frac{1}{8}\right)^2$

There are others!

b. 0.001

Possible answers:  $10^{-3}$ ,  $\left(\frac{1}{10}\right)^3$ ,  $\frac{1}{10^3}$ ,  $\frac{1}{1000}$ ,  $(0.1)^3$



### Activity 2 [Page 2.3]

1. Write the following in scientific notation. Use the TNS activity to check your answers.

a. 345,000

b. 0.0007902

c. 3.14159

Answers: a.  $3.45 \times 10^5$ , b.  $7.902 \times 10^{-4}$ , c.  $3.14159 \times 10^0$

2. Each of the following problems involves very large or small numbers.

a. Human hair grows at a rate  $1.98 \times 10^{-4}$  meters/hour. How long does human hair grow in a year?

Answer: A year has 365 days and 24 hours in each day, so a year is  $365 \times 24 = 8760 = 8.76 \times 10^3$  hours long. Human hair will grow  $(8.76 \times 10^3)(1.98 \times 10^{-4}) = 17.3448 \times 10^{-1} \approx 1.7345 \times 10^0$ , or approximately 1.7345 meters in a year.

b. Juno sent a message from its orbit around Jupiter (540,000,000 miles away) in July 2016. Radio waves travel at the speed of light, which is 186,000 miles per second. How long did the message take to make the trip? Express your answer in seconds using scientific notation. How many minutes is this?

Answer:  $\frac{5.4 \times 10^8}{1.86 \times 10^5} = 2.90322 \times 10^3$  seconds. This is approximately 2,903 seconds and there are

60 seconds in a minute, so it takes approximately  $\frac{2903}{60} \approx 48.4$  minutes for the message to reach Earth.



- c. One mole of hydrogen has a mass of 1.008 grams. If there are approximately  $6.022 \times 10^{23}$  hydrogen atoms in one mole, what is the approximate mass of one hydrogen atom?

Answer:  $\frac{1.008 \times 10^0}{6.022 \times 10^{23}} \approx 0.1674 \times 10^{-23} \approx 1.674 \times 10^{-24}$  grams.

- d. The United States' budget deficit in 1998 was  $\$5.653 \times 10^{12}$ . The budget deficit is projected to be 19.3 trillion dollars at the end of 2016. How many times larger will the budget deficit be then, compared to 1998?

Answer: \$19.3 trillion is  $\$19,300,000,000,000 = \$1.93 \times 10^{13}$ , so the deficit at the end of 2016 will be  $\frac{1.93 \times 10^{13}}{5.653 \times 10^{12}} \approx 0.3414 \times 10^1 \approx 3.414 \times 10^0$ , or roughly three and half times larger than the budget deficit in 1998.