Building Concepts: Creating Equivalent Fractions

TEACHER NOTES

Lesson Overview

The numerator and denominator of a fraction can be multiplied by a common factor without changing the position of the fraction on a number line or the amount of shaded area in a unit square. So, multiplying the numerator and denominator of a fraction by a common factor creates an equivalent fraction.

The fraction $\frac{n}{n}$ ($n \neq 0$) is the same as the number 1, and multiplying any number by 1 does not change the value of the number.

Learning Goals

1. Express that a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{(n \times a)}{(n \times b)}$;
2. Generate a fraction equivalent to a given fraction but with a smaller numerator and denominator;
3. Generate a fraction equivalent to a given fraction but with a larger numerator and denominator.

Prerequisite Knowledge

Creating Equivalent Fractions is the fourth lesson in a series of lessons that explore fractions. This lesson builds on the concepts explored in the previous lessons: *What is a Fraction? Equivalent Fractions*, and *Fractions and Unit Squares*. Students should be familiar with the terms *unit fraction*, *equivalent fractions*, and *improper fraction*. Prior to working on this lesson students should understand:

- two fractions are considered equivalent if they are located at the same point on a number line or represent the same amount of shaded area in a unit square.
- unit squares and number lines can be used to show equivalent fractions.

Vocabulary

- **factors:** numbers multiplied together to get another number (the product).
- **common divisor**: a number that divides both a and b in the fraction $\frac{a}{b}$ without leaving a remainder.

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

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Building Concepts: Creating Equivalent Fractions

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Creating Equivalent Fractions_Student.pdf
- Creating Equivalent Fractions_Student.doc
- Creating Equivalent Fractions.tns
- Creating Equivalent Fractions_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

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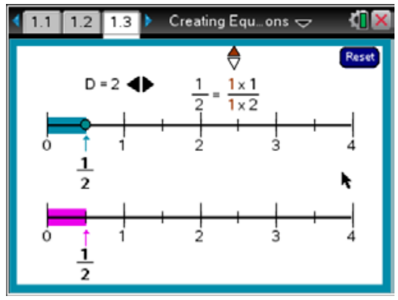
Mathematical Background

This TI-Nspire™ activity helps students understand how equivalent fractions are created. If the numerator and denominator have a common factor, the common factor can be divided out to create an equivalent fraction. This is because the fraction $\frac{n}{n}$ ($n \neq 0$) is the same as the number 1, and multiplying any number by 1 does not change the value of the number. The formal mathematical justification comes from the fact that n and $\frac{1}{n}$ are multiplicative inverses, whose product is always 1. The number 1 is the multiplicative identity, and $1\boldsymbol{a} = \boldsymbol{a}$ for any value of $\boldsymbol{a}$. Two approaches can be used: multiplying (or dividing) both numerator and denominator by a common factor or multiplying the fraction $\left(\frac{a}{b}\right)$ by the fraction $\left(\frac{n}{n}\right)$. This lesson is intended to be a deeper investigation into equivalent fractions, going beyond recognizing and identifying them as students did in the earlier lesson *Equivalent Fractions*. Students should know that two numbers are equivalent (equal) if they are located at the same point on the number line and that the larger of two numbers is the one farther to the right on the number line. To create equivalent fractions with a larger numerator and denominator students multiply both by a number greater than 1. To create an equivalent fraction with a smaller numerator and denominator (i.e., reducing a fraction) students divide both by a nonzero number that is a common factor of both the numerator and denominator. (Note that $\frac{\left(\frac{1}{b}\right)}{\left(\frac{1}{b}\right)}$ still has value 1).

Part 1, Page 1.3

Focus: Students will generate equivalent fractions by multiplying by a number $\frac{n}{n}$.



Page 1.3 shows two number lines. The horizontal arrows next to D set the denominator of a fraction. Drag the dot along the number line to set a numerator for the fraction with denominator D. The vertical arrows near the top can be used to choose a value to multiply both the numerator and denominator of the fraction.

TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Teacher Tip: Have students work in pairs. Instruct one student to write a fraction then have the second student use the number lines on page 1.3 to find an equivalent fraction. Have students explain how they found their answers. Let students take turns finding the equivalent fraction.

Class Discussion

Have students…	Look for/Listen for…
• ***Leave D set at 2. Select the vertical arrow at the top right. Describe the changes in the expression below the arrow and on the two number lines.***	Answer: The number multiplying the numerator and denominator of $\frac{1}{2}$ changes, as do the partitions on the bottom number line and the fraction on the bottom number line that is equivalent to $\frac{1}{2}$. The top number line does not change.

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Class Discussion (continued)

- ***Use the vertical arrows at the top of the page to select 5. Looking at the page, which of the following statements is true? Explain your reasoning.***

  a. ***5 copies of*** $\frac{1}{10}$ ***is the same as 1 copy of*** $\frac{1}{2}$***.***

  b. $\frac{5}{10}$ ***is equivalent to*** $\frac{1}{2}$***.***

  c. $\frac{5}{10}$ ***is larger than*** $\frac{1}{2}$***.***

  d. $\frac{5}{10}$ ***is smaller than*** $\frac{1}{2}$***.***

  Answer: a, b are true; c and d are false. ($\boldsymbol{\frac{5}{10}}$ is another name for $\boldsymbol{\frac{1}{2}}$).

- ***Find four different equivalent fractions for:***

  a. $\frac{3}{4}$

  Answer: $\frac{3}{4} = \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \ldots$

  b. $\frac{8}{12}$

  Answer: $\frac{8}{12} = \frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{10}{15}, \frac{12}{18}, \ldots$

  c. $\frac{3}{1}$

  Answer: $\frac{3}{1} = \frac{6}{2}, \frac{9}{3}, \frac{12}{4}, \frac{15}{5}, \frac{18}{6}, \ldots$

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Class Discussion (continued)

- ***Make a conjecture about whether each of the following is possible. Write Possible or Impossible. Check your conjecture using the number line. Using only whole numbers in the numerator and denominator, you can find a fraction equivalent to***

    a. $\frac{4}{5}$ ***with a denominator of 12.***

    Answer: Impossible.

    b. $\frac{4}{5}$ ***with a numerator of 12***

    Answer: Possible, $\frac{12}{15}$.

    c. $\frac{5}{3}$ ***with a denominator of 18***

    Answer: Possible, $\frac{30}{18}$.

    d. $\frac{7}{6}$ ***with a denominator of 10***

    Answer: Impossible.

    e. $\frac{20}{8}$ ***with a denominator of 4***

    Answer: Possible, $\frac{10}{4}$.

> **Teacher Tip:** Remind students that they made conjectures in the previous lesson *Fractions and Unit Squares*. Reinforce their understanding that a conjecture is a conclusion they have drawn based on given information.

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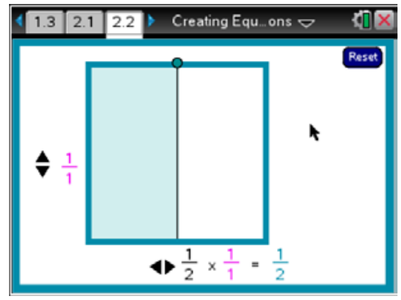
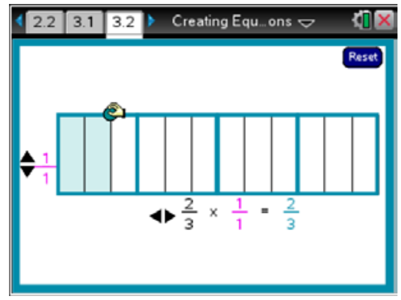
Part 2, Pages 2.2 and 3.2

Focus: Students will use multiplication and area models to generate equivalent fractions. Students learn that equivalent fractions can be generated by multiplying a given fraction by a fraction equal to 1.





Page 2.2 shows a unit square with one half shaded. The arrows at the bottom of the page set a unit fraction and display equal partitions of the unit square. Drag the dot on the top of the square to set the numerator of the fraction and to shade in the area represented by the fraction. The vertical arrows on the left side of the page multiply both the numerator and denominator of the fraction by $\frac{n}{n}$ and display the corresponding partitions of the unit square. To reset the page, select **Reset** in the upper right corner.

Page 3.2 can be used to find equivalent improper fractions, using the same approach as that on page 2.2.

> **Teacher Tip:** Have students identify the factors in the multiplication at the bottom of the page. Help students identify which part of the interactive model represents the factors and which part shows the product. Reinforce that the product is the equivalent of the first fraction in the multiplication.

As students explore multiplying fractions to create equivalent fractions, encourage them to make predictions and explain their reasoning.

Class Discussion

Have students…

Use the bottom arrows and drag the dot to create the fraction $\frac{5}{6}$.

- ***Use the arrows on the left to multiply*** $\frac{5}{6}$ ***by*** $\frac{2}{2}$. ***What is the result and how does the picture support the result?***

  Answer: The six vertical rectangles were divided horizontally into two parts, resulting in 12 partitions of the unit square. 10 of the 12 rectangles are shaded.

- ***Make a conjecture about what the picture will look like when you multiply by*** $\frac{4}{4}$.

  Possible answer: The six vertical rectangles will be divided horizontally into four parts, making 20 rectangles shaded out of 24.

Look for/Listen for…

Class Discussion (continued)

- ***Check your conjecture using the number line.***

  Possible answer: I was right.

- ***If you multiply $\frac{5}{6}$ by $\frac{12}{12}$, how many small rectangles will tile the unit square? Why?***

  Answer: 72, because that is the product of 6 and 12.

- ***If you multiply $\frac{5}{6}$ by $\frac{7}{7}$, make a conjecture about the numerator of the result. Check your conjecture using the number line.***

  Answer: The numerator will be 35.

Write the answer for each of the following:

- ***Find a fraction equivalent to $\frac{3}{7}$ that has a denominator of 28.***

  Answer: $\frac{12}{28}$

- ***Find a fraction equivalent to $\frac{5}{11}$ that has a numerator of 25.***

  Answer: $\frac{25}{55}$

✓ ***To find a fraction equivalent to $\frac{7}{10}$, how many small rectangles will tile the unit square? Explain your reasoning.***

(Question #1 on the Student Activity sheet.)

Possible answer: In order to get a fraction equivalent to $\frac{7}{10}$, you would need to cover the same area that is covered by that fraction on the unit square. Since $\frac{14}{20}$ is equivalent to $\frac{7}{10}$, 280 small rectangles would tile $\frac{7}{10}$ of the unit square.

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Class Discussion (continued)

On Page 3.2, create the fraction $\frac{11}{6}$.

- ***How many copies of $\frac{1}{6}$ are shaded?***

  Answer: 11.

- ***Multiply by $\frac{2}{2}$. How many rectangles are shaded?***

  Answer: 22.

What is the area of each rectangle? Explain your reasoning.

Possible answer: Each has area $\frac{1}{12}$ of a unit square because each unit square is now partitioned into 12 rectangles that have the same area.

- ***What fraction is represented by the shaded rectangles?***

  Answer: $\frac{22}{12}$.

Have students…

- ***Find two equivalent fractions for $\frac{19}{8}$ and explain how the pictures are used to illustrate that the fractions are equivalent to $\frac{19}{8}$.***

Look for/Listen for…

Answer: $\frac{38}{16}$; the vertical rectangles representing $\frac{19}{8}$ are cut in half so there are now 38 copies of $\frac{1}{16}$ shaded. $\frac{57}{24}$; the vertical rectangles representing $\frac{19}{8}$ are cut in thirds so there are now 57 copies of $\frac{1}{24}$ shaded.

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Class Discussion (continued)

Make a conjecture about what value goes in the blank. Check your conjecture using the number line from either pages 1.3, 2.2, or 3.2.

- $\frac{\square}{5} = \frac{28}{20}$

  Answer: 7.

- $\frac{\square}{5} = \frac{24}{20}$

  Answer: 6.

- $\frac{6}{7} = \frac{\square}{28}$

  Answer: 24.

Have students…

Look for/Listen for…

Identify each statement as true or false and explain your reasoning.

- $\frac{9}{4}$ ***is equivalent to*** $\frac{3}{2}$.

  Answer: False. No one number will multiply to make 2 into 4 and 3 into 9.

- $\frac{10}{12}$ ***is equivalent to*** $\frac{5}{6}$.

  Answer: True. $\frac{2}{2}$ will make $\frac{5}{6}$ into $\frac{10}{12}$ because $\mathbf{2 \times 5 = 10}$ and .

- $\frac{4}{5}$ ***is equivalent to*** $\frac{16}{25}$.

  Answer: False. $\mathbf{5 \times 5 = 25}$ is 25 but $\mathbf{5 \times 4}$ is not 16.

Give an example that either supports the claim or shows that it is wrong.

- ***Multiplying any number by 1 does not change the value of the number.***

  Possible answer: True. $1 \times 8 = 8$.

- ***A fraction with a numerator equal to its denominator will always be equivalent to 1.***

  Answer: True. $\boldsymbol{\frac{6}{6}} = 1$ (this is the multiplicative inverse or reciprocal property).

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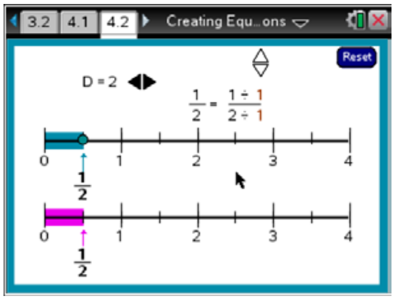
Class Discussion (continued)

✓ ***Multiplying a fraction by*** $\frac{2}{2}$ ***will double the size of the fraction.***
(Question #2 on the Student Activity sheet.)

Answer: False. It will make an equivalent fraction, not change the value because $\frac{2}{2}$ is equal to 1. For example, multiplying $\frac{2}{3} \times \frac{2}{2}$ does not double the product to $\frac{4}{3}$. The result is $\frac{4}{6}$, which is equivalent to $\frac{2}{3}$.

Part 3, Page 4.2

Focus: Students learn how to create equivalent fractions by dividing the numerator and denominator of a fraction by another fraction equal to 1.



Page 4.2 behaves like page 1.3 but the arrows on the top left divide both numerator and denominator of the fraction by a common divisor, which allows students to reduce fractions.

The values for the divisors begin with 1, cycle through the common factors in order of magnitude, and end with the original denominator, which does not produce an equivalent fraction unless the original fraction was a whole number. Sometimes you can find equivalent fractions with smaller numerators and denominators than the original. This is called reducing a fraction. Have students observe several examples of reducing fractions. Then lead them in a discussion of their observations.

> **Teacher Tip:** Once students understand the concept of reducing fractions have them discuss how multiplying to create equivalent fractions is similar to dividing to create equivalent fractions. Then have students identify the ways in which the procedures differ.

Class Discussion

- ***Find a fraction equivalent to $\frac{30}{9}$ whose denominator and numerator are smaller than 30 and 9. What divisors produced these fractions?***

  Answer: $\frac{10}{3}$, divided by 3.

- ***Find all the fractions equivalent to $\frac{30}{12}$ whose denominator and numerator are smaller than 30 and 12. What divisors produced these fractions?***

  Answer: $\frac{15}{6}$, divided by 2; $\frac{10}{4}$, divided by 3; $\frac{5}{2}$, divided by 6.

- ***Find all the fractions equivalent to $\frac{15}{8}$ whose denominator and numerator are smaller than 15 and 8. What divisors produced these fractions?***

  Answer: There are none.

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Class Discussion

- ***Look at the divisors and the original numerator and denominator carefully. What observations can you make about the relationship in each of the problems above?***
  Answer: If the numerator and denominator of a fraction have a common factor, you can find an equivalent fraction that has a smaller numerator and denominator. If they don't have a common factor, you cannot.
- ***Suppose you had to reduce*** $\frac{24}{16}$***. Think about your answers above and make a conjecture about the divisors that will work. Check your thinking using the number line.***

  Answer: 2, 4, and 8.

✓ ***Suppose you want to reduce*** $\frac{25}{10}$***. Why is it not useful to try 2 as a divisor?***

(Question #3 on the Student Activity sheet.)

Answer: Because you need a number that is a factor of both the numerator and denominator. 2 is only a factor of the denominator.

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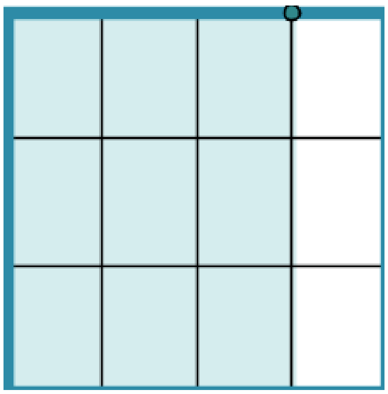
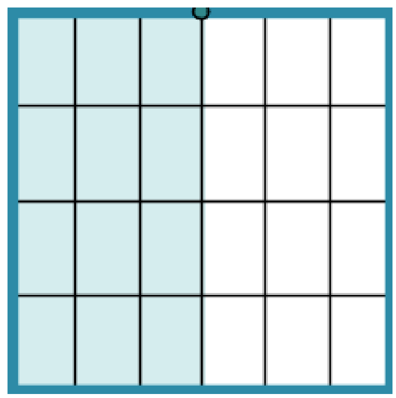
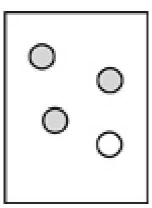
Sample Assessment Items

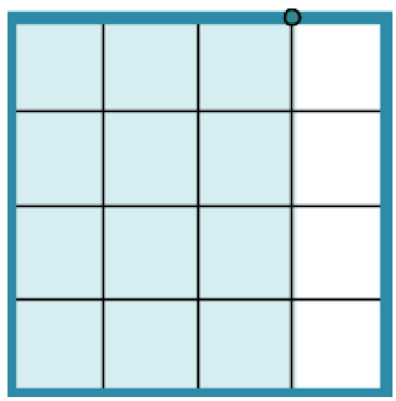
After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Which display shows that $\frac{9}{12}$ is equivalent to $\frac{3}{4}$?

a. 

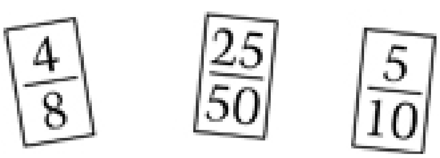
b. 

c. 

d. 

Answer: a.

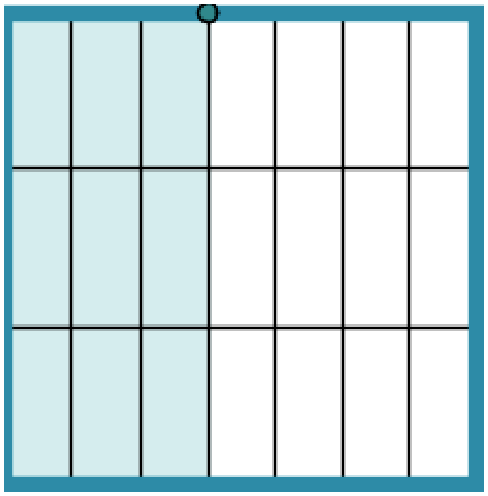
2. These three fractions are equivalent. Give <u>two</u> more fractions that are equivalent to these.



NAEP 2007, Grade 4

Possible answer: $\boldsymbol{\frac{6}{12}, \frac{8}{16}, \frac{30}{60}, \ldots}$

3.



What fraction of the figure above is shaded?

a. $\frac{9}{7}$ b. $\frac{3}{7}$ c. $\frac{1}{3}$ d. $\frac{9}{3}$ e. $\frac{4}{7}$

Answer: b.

4. Use the numbers 15, 12, 10, 9, 7, 6, 5 in the blanks to make true statements.

a. $\frac{5}{4} = \frac{\square}{\square}$ ***Possible answer:*** $\boldsymbol{\frac{5}{4} = \frac{15}{12}}$

b. $\frac{3}{\square} = \frac{\square}{21}$ ***Possible answer:*** $\boldsymbol{\frac{3}{7} = \frac{9}{21}}$

c. $\frac{12}{\square} = \frac{6}{\square}$ ***Possible answer:*** $\boldsymbol{\frac{12}{10} = \frac{6}{5}}$

d. $\frac{12}{20} = \frac{\square}{\square}$ ***Possible answer:*** $\boldsymbol{\frac{12}{20} = \frac{6}{10}}$

Student Activity solutions

Vocabulary

factors:

numbers multiplied together to get another number (the product)

common divisor:

a number that divides both a and b in the fraction $\frac{a}{b}$ without leaving a remainder

In this activity, you will use common factors and divisors to create equivalent fractions.

1. To find a fraction equivalent to $\frac{7}{10}$, how many small rectangles will tile the unit square? Explain your answer.

   Possible answer: In order to get a fraction equivalent to $\frac{7}{10}$, you would need to cover the same area that is covered by that fraction on the unit square. Since $\frac{14}{20}$ is equivalent to $\frac{7}{10}$, 280 small rectangles would tile $\frac{7}{10}$ of the unit square.

2. Is the following claim true or false? Explain your reasoning, and then give an example.
   Multiplying a fraction by $\frac{2}{2}$ will double the size of the fraction.

   Answer: False, It will make an equivalent fraction, not change the value because $\frac{2}{2}$ is equal to 1. For example, multiplying $\frac{2}{3} \times \frac{2}{2}$ does not double the product to $\frac{4}{3}$. The result is $\frac{4}{6}$, which is equivalent to $\frac{2}{3}$.

3. Suppose you want to reduce $\frac{25}{10}$. Why is it not useful to try 2 as a divisor?

   Answer: Because you need a number that is a factor of both the numerator and denominator. 2 is only a factor of the denominator.

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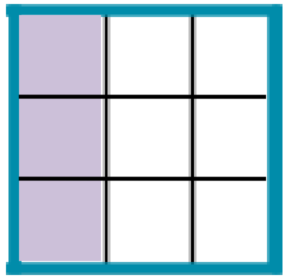
4. *The fraction $\frac{1}{3}$ is multiplied by $\frac{1}{3}$. What is the area of each rectangle in the product? Draw an area model to illustrate the multiplication. Explain your answer.*



Answer: Each rectangle has area $\frac{1}{9}$ of a unit square because the unit square is now partitioned into 9 rectangles that have the same area.