Building Concepts: Adding Ratios

Lesson Overview

In this TI-Nspire™ lesson, students continue their investigation of ratios, exploring graphical representations of the sum of two ratios. They build upon their knowledge that ratios represent the relationship between two (or more) related quantities; and that, since the values in a ratio are quantities, they can be combined to form a new ratio.

The sum of two equivalent ratios is an equivalent ratio and lies on the same line as the original ratios. The sum of two ratios that are not equivalent lies on a line that is between the lines formed by the two ratios.

Learning Goals

1. Recognize that you can add ratios by adding corresponding values;
2. recognize that adding equivalent ratios results in a ratio that is equivalent to the original ratio and that adding non-equivalent ratios results in a ratio that is different from both of the original ratios;
3. recognize that a common trend between pairs of ratios can be reversed when the groups are combined (Simpson's Paradox).

Prerequisite Knowledge

Adding Fractions is the thirteenth lesson in the series of lessons that investigates ratios and proportional relationships. The lesson builds on students' knowledge of proportions and their graphs. Prior to working on this lesson students should have completed *Connecting Ratios to Graphs*, *Connecting Ratios to Equations, Proportional Relationships,* and *Solving Proportions*. Students should:

- understand how to find a unit rate;
- be able to distinguish situations that involve proportional relationships from those that do not.

Vocabulary

- **proportional relationship**: a collection of equivalent ratios.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Adding Ratios_Student.pdf
- Adding Ratios_Student.doc
- Adding Ratios.tns
- Adding Ratios_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice, and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

In this TI-Nspire™ lesson, students continue their investigation of ratios, exploring graphical representations of the sum of two ratios. They build upon their knowledge that ratios represent the relationship between two (or more) related quantities. As one of the quantities increases or decreases by a factor, so does the other. Because the values in a ratio are quantities, they can be combined to form a new ratio. For example, 5 hits in 7 at bats could be combined with 3 hits in 6 at bats to form a new ratio: 8 hits in 13 at bats. However, the sum of the fractions associated with the ratios ($\frac{5}{7}+\frac{3}{6}$) is not the same as the fraction associated with the sum of the ratios ($\frac{8}{13}$). Ratios represent a pair of numbers and can be plotted as an ordered pair in a coordinate plane, but the fraction associated with a ratio represents the slope of the ray beginning at the origin containing that ordered pair.

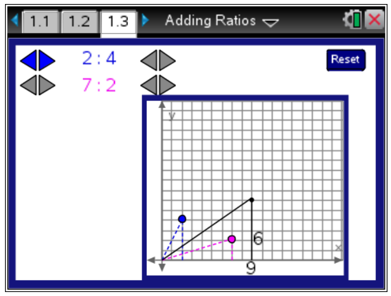
If two equivalent ratios are added, the result will be a new ratio that is equivalent to those added; if *a:b* is added to *ca:cb* for some positive number *c*, the result will be $(a+ca):(b+cb)$, which will be $a(1+c):b(1+c)$, where the values in the original ratio *a:b* have been multiplied by the common positive number $(1+c)$.

A phenomenon that occurs often enough to be given its own name is called *Simpson's Paradox* in which a correlation (trend) present in different groups is reversed when the groups are combined.

Part 1, Page 1.3

Focus: What is the graphical representation when you add two ratios?

On page 1.3, the arrows at the top of the page set two ratios and display a segment of the line that represents a set of equivalent ratios. The pink and blue dots can be moved by dragging or using the arrows on the keypad. Note a third black triangle is drawn with horizontal and vertical sides that correspond to the sum of the values of the coordinates of the pink and blue points.



TI-Nspire Technology Tips

Use the tab key to toggle between the values in each ratio.

To reset the document, press **Reset** or ctrl del.

The following questions focus on graphing the sum of two ratios and noting that the sum of two equivalent ratios is an equivalent ratio and lies on the same line as the original ratios. The sum of two ratios that are not equivalent lies on a line that is between the lines formed by the two ratios.

Class Discussion

Use Reset to make sure the graph is at the original position.

- ***What are the coordinates of the three points on the line segments?***

  Answer: (2, 4); (7, 2); (9, 6)

- ***What ratio is associated with each of the line segments?***

  Answer: 2:4 is the blue, 7:2 is the pink, and 9:6 is the black.

- ***How are the ratios related?***

  Answer: The sum of the corresponding values in the blue and pink ratios makes the black ratio.

Make one ratio 2:4 and the other 5:10.

- ***How are 2:4 and 5:10 related? Explain your reasoning.***

  Answer: They are equivalent ratios because dividing both 2 and 4 by 2 produces the ratio 1:2 and dividing 5 and 10 by 5 produces the ratio 1:2.

- ***What do you observe about the points? Give a reason for your answer.***

  Answer: All of the points lie on the same line. This is because 7:14 is also equivalent to 1:2 (dividing the values by 7) so 7:14, 2:4 and 5:10 belong to the same collection of equivalent and thus lie on the same straight line.

Student Activity Questions—Activity 1

1. **Set one ratio at 1.5:5. Find a second ratio such that the sum of the two ratios determines a point on the specified line. Explain your reasoning in each case.**

   a. **line determined by 1.5:5**

   Answer: Any ratio equivalent to 1.5:5. For example, 3:10 because then all three ratios are equivalent and so lie on the same line.

   b. **line with slope 2**

   Possible answer: Any ratio that produces a sum equivalent to 1:2. For example, 4.5:7 and 1.5:5 will produce the ratio 6:12, which will lie on a line with rate of change of $\frac{12}{6}$ or 2.

   c. **line *y* = *x***

   Possible answer: Any ratio that sums with 1.5:5 to produce a ratio 1:1. For example, 1.5:5 and 4.5:1 will add to 6:6, which will determine the line with rate of change of 1.

2. **Given the ratio: 4.5:2, find another ratio so that the sum of the two ratios is not between the lines associated with each ratio.**

   Answer: The only ratios such that the sum of the ratios will not lie on a line between the lines produced by each individual ratio will be an equivalent ratio. So the answer might be 9:4 or 13.5:6.

3. **Consider two ratios: 2:7 and 3:8. (Note that you may want to use the TNS files from your work with connecting ratios to graphs and proportions to support your thinking for these questions.)**

   a. **Which ratio produces a steeper line? Show how you found your answer.**

   Possible answer: Make the first value in each ratio 6, so you would have 6:21, which is associated with the rate of $\frac{21}{6}$ and 6:16, which is associated with the rate $\frac{16}{6}$. The rate $\frac{21}{6}$ rises 21 vertical units for every 6 over, while the rate $\frac{16}{6}$ rises 16 units for every 6 over, so the line associated with the ratio 2:7 will be steeper.

Student Activity Questions—Activity 1 (continued)

b. Given two ratios, how can you tell which will be associated with a steeper line? Explain your reasoning and use the TNS file to give an example that supports your thinking.

Answer: If two ratios have the same first value, the line associated with the ratio that has the larger second value will be steeper because for the same horizontal movement, you will have a larger vertical movement to generate that line. So, for 3:2 and 3:7, the associated rates would be $\frac{2}{3}$ and $\frac{7}{3}$. In both cases you go over 3, but in the first you only move vertically 2 units and in the second you move vertically 7 units, so the second ratio will be associated with a steeper line. If two ratios have the same second value, the ratio with the smaller first value will be associated with the steeper line because you will have to move farther horizontally for the larger second value for the same vertical movement. For example, consider 2:3 and 5:3. In the first case the rate is $\frac{3}{2}$; you go over 2 units and up 3. For 5:3, the rate is $\frac{3}{5}$; you go over 5 units for the same vertical rise. So the first ratio with the smaller first value will be steeper.

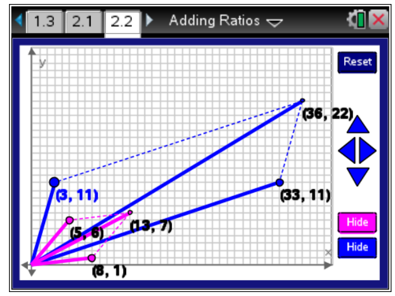
> **Teacher Tip:** The following question asks students to think about the steepness of the lines generated by adding two ratios.

4. Make a conjecture about when the line *y* =*x* bisects the lines formed by the two ratios, *a:b* and *c:d*.

Sample answer: A good conjecture might be when $c = b$ and $d = a$. One proof depends on reasoning about angles formed by parallel lines and angles in triangles, which comes in later in geometry. Another is to show that the completed figure will be a kite and use the fact that diagonals of a kite bisect each other at right angles, again using knowledge from more formal geometry work.

Part 2, Page 2.2

Focus: Comparing sums of pairs of ratios.



Page 2.2 displays two sets of ratios, like the one shown on page 1.3; the pink segments determined by two ratios and their sum and the blue segments determined by two other ratios and their sum.

Change the ratios by moving each of the points along the graph. Do this by dragging points on the graph or using the arrows on the page or the arrow keys on the handheld.

Use the tab key to toggle control between the pink and the blue points. Hide either the pink or the blue segments by using the **Hide** button on screen or the menu key.

In the following questions, students investigate different pairs of ratios within the same context. They consider the phenomena that in some cases, the trend indicated by the ratios for a certain outcome can be greater than another in two different groups in the same context, but when the groups are combined, the trend is reversed. (This is an example of what is known as Simpson's Paradox).

> **Teacher Tip:** At the end of the lesson, have students change the ratios for the blue and pink segments and explain what each segment represents. Discuss as a class the effects the rates associated with the two ratios have on the steepness of the segments.

Class Discussion

Consider the display on page 2.2. Use the Hide button to display only one set of segments. What do you think each represents?

- ***the three blue segments***

  Answer: The sum of the ratios 3:11 and 33:11 is 36:22

- ***the three pink segments***

  Answer: The sum of the ratios 8:1 and 5:6 is 13:7.

Student Activity Questions—Activity 2

1. **Display both sets of segments on page 2.2. Explain which is steeper in each case.**

   a. **the blue segment representing the ratio 33:11 or the pink one representing the ratio 8:1**

   Answer: the blue segment because the rate or constant of proportionality is larger; $\frac{1}{3}$ is larger than $\frac{1}{8}$, and you can tell from the graph because you go farther horizontally for a rate of $\frac{1}{8}$ than for a rate of $\frac{1}{3}$ for the same vertical movement of 1 unit.

   b. **the blue segment representing the ratio 3:11 or the pink one representing the ratio 5:6**

   Answer: the blue segment because you can tell from the graph. The rate is larger; $\frac{11}{3}$ $(3\frac{2}{3})$ is larger than $\frac{6}{5}$ $(1\frac{1}{5})$.

   c. **the blue segment representing the sum of the blue ratios or the pink segment representing the sum of the pink ratios**

   Answer: the blue segment because you can tell from the graph and because the rate is larger; $\frac{22}{36}$ is larger than $\frac{7}{13}$.

   d. **Does your answer to part c seem reasonable? Why or why not?**

   Possible answer: Yes, because if one ratio is larger than another and a third ratio is larger than a fourth, it is reasonable that the sum of the first and third is larger than the sum of the second and fourth ratios.

2. **Refer to the grid on page 2.2. Change the point representing the ratio 5:6 to a point representing the ratio 20:24. Explain which is steeper in each case:**

   a. **the blue segment representing the ratio 33:11 or the pink one representing the ratio 8:1**

   Answer: This is the same answer as 1a. The blue segment is larger because the rate or constant of proportionality is larger; $\frac{1}{3}$ is larger than $\frac{1}{8}$ and you can tell from the graph.

   b. **the blue segment representing the ratio 3:11 or the pink one representing the ratio 20:24**

   Answer: the blue segment because you can tell from the graph. The rate is larger; $\frac{11}{3}$ $(3\frac{2}{3})$ is larger than $\frac{24}{20}$ or $\frac{6}{5}$ $(1\frac{1}{5})$.

Student Activity Questions—Activity 2 (continued)

c. **the blue segment representing the sum of the blue ratios or the pink segment representing the sum of the pink ratios**

Answer: the pink segment because you can tell from the graph and because the rate is larger; $\frac{28}{25}$ is larger than $\frac{22}{36}$.

d. **Does your answer to part c seem reasonable? Why or why not?**

Possible answer: It seems strange that the sums have the reverse relationship in terms of size from each of the two parts.

Class Discussion

Suppose you have two baseball players A and B.

- ***The table below shows the number of hits and outs for the first and second half of a season for both players***.

Season 1	***First Half***			***Second Half***			***Season H/O***
	Hits	Outs	Hits/Outs	Hits	Outs	Hits/Outs	
Player A	3	11	$\frac{3}{11}$	33	11	$\frac{33}{11}$	$\frac{36}{22}$
Player B	5	6	$\frac{5}{6}$	8	1	$\frac{8}{1}$	$\frac{13}{7}$

How do the numbers relate to the problems above? Explain how player A and player B compare over the two halves of the season and for the whole season.

Answer: Player A is like the blue ratios and player B is like the pink ratios in problem 8. In both halves of the season, player B has a better ratio of hits to outs and a better ratio for the whole season.

- ***The table below shows the number of hits and outs for a second season for the same two players. Answer the question above for the new season.***

Season 2	***First Half***			***Second Half***			***Season H/O***
	Hits	Outs	Hits/Outs	Hits	Outs	Hits/Outs	
Player A	3	11	$\frac{3}{11}$	33	11	$\frac{33}{11}$	$\frac{36}{22}$
Player B	20	24	$\frac{20}{24}$	8	1	$\frac{8}{1}$	$\frac{28}{25}$

Answer: Player A is like the blue ratios and player B is like the pink ratios in the graph on page 2.2 of the TNS activity. In both halves of the season, player B has a better ratio of hits to outs, but player A has better ratio for the whole season.

Additional Discussion

On page 2.2 of the TNS activity change the point representing the ratio 5:6 to a point representing the ratio 20:24.

- ***Which line is steeper: the blue segment representing the sum of the blue ratios or the pink segment representing the sum of the pink ratios?***

  Answer: You can tell from the graph that the sum of the pink ratios determines a segment that is steeper than the sum of the blue ratios. For example, when $x = 8$, the associated vertical height, the *y*-coordinate, for the pink sum is larger than the y-coordinate for the blue sum. You can also compare the two slopes. The sum of the pink ratios is 28:25 and associated with the point (28, 25) and the slope for the segment is $\frac{25}{28}$. The sum of the blue ratios remains the same as above, 36:22, and determines the point (36, 22). The slope of the pink segment is $\frac{25}{28}$, which is larger than the slope for the blue segment which is $\frac{22}{36}$. One way to compare the slopes is to use ratio tables:

28	5.6	11.2	1.12	12.32
25	5	10	1	11

- ***The phenomenon described in the previous problem is called Simpson's Paradox. You might want to do an internet search about real-life situations involving Simpson's Paradox.***

  Answer: Students might give short reports on what they find in their search.

- ***The middle and high schools in a district were hiring new teachers. The number of males and females who were hired and who applied but not hired is given in the table.***

	Middle School	***High School***	***Both schools***
Men	Hired: 17 Not hired: 4	Hired: 7 Not hired: 12	Hired: 24 Not hired: 16
Women	Hired: 8 Not hired: 1	Hired: 7 Not hired: 10	Hired: 15 Not hired: 11

Additional Discussion (continued)

Have students…	Look for/Listen for…
• ***Compare the number of men and women the middle school and the high school hired. Explain your reasoning.***	Possible answers: Some may use absolute numbers: The middle school hired 8 women and 17 men, while the high school hired 7 women and 7 men, so both departments hired the same or fewer women than men. Others may argue that in the middle school almost all of the women who applied got hired but just over $\frac{3}{4}$ of the men got hired, while in the high school just over one half of the men who applied got hired and more than half of the women who applied got hired. Still others might note that while more men overall got hired, more men applied as well (24 out of 40).
• ***Use the TNS activity to graph the segments associated with each ratio. If you interpret the results across the two departments, were more men who applied hired than women who applied? Explain your reasoning.***	Answer: Think about the unit rate for those hired per those not hired for each ratio. In the middle school, 4.25 men were hired for each one not hired while 8 women were hired to each one not hired. In the high school 0.7 women were hired for each one not hired and 0.55 men were hired for each one not hired. In both of these cases, there were more women hired per those that applied than men. So if you were a woman and applied at ether school, you were more likely to get hired than if you were a man and applied. But overall, 1.5 men were hired for each one that applied and about 1.37 women were hired for each one that applied, so a man that applied was more likely to be hired.
• ***Is this an example of Simpson's Paradox? Why or why not?***	Answer: It is an example because each of the departments hired a greater ratio of men to those who applied than of women but together the ratio for women who got hired was greater.

Additional Discussion (continued)

Have students…	Look for/Listen for…
• ***Do you think it is harder to get a job in the middle school or the high school? Explain your reasoning.***	Answer: The middle school is hiring more people overall than the high school with 25 people compared to 14. The success rate was higher in the middle school with 25 hires to 5 not hired as compared to 14 hired in the high school to 22 not hired. That indicates that 5 people were hired in the middle school for every 1 not hired while $\frac{7}{11}$ or about 0.63 people were hired in the high school for everyone that was not hired.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. The ratio 50:100 is equivalent to the ratio 1:2 and the ratio is 75:100 is equivalent to the ratio 3:4. Is the sum of the ratios 50:100 and 75:100 equivalent to the sum of the ratios 1:2 and 3:4? Why or why not

   Answer: One sum is the ratio 4:6 and the second is 125:200, which is equivalent to 5:8. The ratios are not equivalent so the sums are not equivalent.

2. Find a value for *x*, $x < 10$ so that the following inequalities are true.

   $$\frac{1}{5} < \frac{x}{8}$$

   $$\frac{6}{8} < \frac{x}{5}$$

   $$\frac{7}{13} > \frac{(x+2)}{13}$$

   Answer: x = 4

3. Which of the following is true?

   a. The sum of two ratios will always be associated with a line that is steeper than the line associated with the ratio 1:1

   b. The sum of two equivalent ratios will be associated with a point that will lie on the same ray as the ray associated with each of the ratios.

   c. $\frac{7}{8} + \frac{5}{12} = \frac{12}{20}$

   Answer: b) The sum of two equivalent ratios will be associated with a point that will lie on the same ray as the ray associated with each of the ratios.

Student Activity Solutions

In these activities you will use graphical representations to show the sum of ratios and to compare the sum of two ratios. After completing each activity, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Set one ratio at 1.5:5. Find a second ratio such that the sum of the two ratios determines a point on the specified line. Explain your reasoning in each case.

   a. line determined by 1.5:5

      Answer: Any ratio equivalent to 1.5:5. For example, 3:10 because then all three ratios are equivalent and so lie on the same line.

   b. line with slope 2

      Possible answer: Any ratio that produces a sum equivalent to 1:2. For example, 4.5:7 and 1.5:5 will produce the ratio 6:12, which will lie on a line with rate of change of $\frac{12}{6}$ *or 2.*

   c. line $y = x$

      Possible answer: Any ratio that sums with 1.5:5 to produce a ratio 1:1. For example, 1.5:5 and 4.5:1 will add to 6:6, which will determine the line with rate of change of 1.

2. Given the ratio: 4.5:2, find another ratio so that the sum of the two ratios is not between the lines associated with each ratio.

   Answer: The only ratios such that the sum of the ratios will not lie on a line between the lines produced by each individual ratio will be an equivalent ratio. So the answer might be 9:4 or 13.5:6.

3. Consider two ratios: 2:7 and 3:8. (Note that you may want to use the TNS lessons from your work with connecting ratios to graphs and proportions to support your thinking for these questions.)

   a. Which ratio produces a steeper line? Show how you found your answer.

      Possible answer: Make the first value in each ratio 6, so you would have 6:21, which is associated with the rate of $\frac{21}{6}$ *and 6:16, which is associated with the rate* $\frac{16}{6}$*. The rate* $\frac{21}{6}$ *rises 21 vertical units for every 6 over, while the rate* $\frac{16}{6}$ *rises 16 units for every 6 over, so the line associated with the ratio 2:7 will be steeper.*

b. Given two ratios, how can you tell which will be associated with a steeper line? Explain your reasoning and use the TNS lesson to give an example that supports your thinking.

Answer: If two ratios have the same first value, the line associated with the ratio that has the larger second value will be steeper because for the same horizontal movement, you will have a larger vertical movement to generate that line. So, for 3:2 and 3:7, the associated rates would be $\frac{2}{3}$ *and* $\frac{7}{3}$*. In both cases you go over 3, but in the first you only move vertically 2 units and in the second you move vertically 7 units, so the second ratio will be associated with a steeper line. If two ratios have the same second value, the ratio with the smaller first value will be associated with the steeper line because you will have to move farther horizontally for the larger second value for the same vertical movement. For example, consider 2:3 and 5:3. In the first case the rate is* $\frac{3}{2}$*; you go over 2 units and up 3. For 5:3, the rate is* $\frac{3}{5}$*; you go over 5 units for the same vertical rise. So the first ratio with the smaller first value will be steeper.*

4. Make a conjecture about when the line $y = x$ bisects the lines formed by the two ratios, $a:b$ and $c:d$.

Sample answer: A good conjecture might be when $c = b$ *and* $d = a$*. One proof depends on reasoning about angles formed by parallel lines and angles in triangles, which comes in later in geometry. Another is to show that the completed figure will be a kite and use the fact that diagonals of a kite bisect each other at right angles, again using knowledge from more formal geometry work.*

Activity 2 [Page 2.2]

1. Display both sets of segments on page 2.2. Explain which is steeper in each case.

a. the blue segment representing the ratio 33:11 or the pink one representing the ratio 8:1

Answer: the blue segment because the rate or constant of proportionality is larger; $\frac{1}{3}$ *is larger than* $\frac{1}{8}$*, and you can tell from the graph because you go farther horizontally for a rate of* $\frac{1}{8}$ *than for a rate of* $\frac{1}{3}$ *for the same vertical movement of 1 unit.*

b. the blue segment representing the ratio 3:11 or the pink one representing the ratio 5:6

Answer: the blue segment because you can tell from the graph. The rate is larger; $\frac{11}{3}$ $(3\frac{2}{3})$ *is larger than* $\frac{6}{5}$ $(1\frac{1}{5})$*.*

c. the blue segment representing the sum of the blue ratios or the pink segment representing the sum of the pink ratios

Answer: the blue segment because you can tell from the graph and because the rate is larger; $\frac{36}{22}$ *is larger than* $\frac{7}{13}$.

d. Does your answer to part c seem reasonable? Why or why not?

Possible answer: Yes, because if one ratio is larger than another and a third ratio is larger than a fourth, it is reasonable that the sum of the first and third is larger than the sum of the second and fourth ratios.

2. Refer back to the grid on page 2.2.Change the point representing the ratio 5:6 to a point representing the ratio 20:24. Explain which is steeper in each case:

a. the blue segment representing the ratio 33:11 or the pink one representing the ratio 8:1

Answer: This is the same answer as 8a. The blue segment is larger because the rate or constant of proportionality is larger; $\frac{1}{3}$ *is larger than* $\frac{1}{8}$ *and you can tell from the graph.*

b. the blue segment representing the ratio 3:11 or the pink one representing the ratio 20:24

Answer: the blue segment because you can tell from the graph. The rate is larger; $\frac{11}{3}$ $(3\frac{2}{3})$ *is larger than* $\frac{24}{20}$ *or* $\frac{6}{5}$ $(1\frac{1}{5})$.

c. Does your answer to part b seem reasonable? Why or why not?

Possible answer: It seems strange that the sums have the reverse relationship in terms of size from each of the two parts.