## , in <br> Building Concepts: Building a Table of Ratios teacher Notes

## Lesson Overview

This TI-Nspire ${ }^{T M}$ lesson has students generate equivalent ratios where tables of those ratios are shown alongside the visual images they used in the first activity, What is a Ratio?. By the end of the lesson, students should understand that the values in each row of the table are a multiple of the values in the original ratio and that the entries in each column can be obtained by adding the original values in the ratio to the previous row.

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Building tables of ratios illustrates that adding any two equivalent ratios produces another equivalent ratio.

## Prerequisite Knowledge

Building a Table of Ratios is the third lesson in a series of lessons that explore the concepts of ratios and proportion. Students should have completed the lessons What is a Ratio? and Introduction to Rates before working on this lesson. Students should understand:

- the concepts of multiplication and division;
- the concepts of ratios and equivalent ratios.


## Learning Goals

1. Identify the role of multiplication and division in the relationship among the entries in a table of ratios;
2. generalize strategies for finding entries in ratio tables;
3. display equivalent ratios in a table;
4. understand the additive approach to generating equivalent ratios.

## Vocabulary

- ratio: a pair of values representing two (or more) quantities that vary in the same relative relationship.
- equivalent ratios: two ratios such that the values in one ratio can be obtained by multiplying both values in the second ratio by a number greater than zero.


## Lesson Pacing

This lesson should take 50 minutes to complete with students, though you may choose to extend, as needed.

## Lesson Materials

- Compatible TI Technologies:

TI-Nspire CX Handhelds, $\quad$ TI-Nspire Apps for iPad $®$, $\quad$ Tl-Nspire Software

- Building a Table of Ratios_Student.pdf
- Building a Table of Ratios_Student.doc
- Building a Table of Ratios.tns
- Building a Table of Ratios_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.


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## Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.


Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice, and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

## Mathematical Background

Equivalent ratios arise by multiplying or dividing each measurement in a ratio pair by the same positive number. For example, 3 meters for every 2 seconds and 9 meters for every 6 seconds are equivalent ratios and are said to have the same ratio.

Tables of ratios build on the earlier work students have done with multiplication tables. At first, when making tables of quantities in equivalent ratios, students may focus on repeated addition to generate equivalent ratios. They might note that adding any two corresponding ratios in the table produces another ratio in the table; i.e., adding any two equivalent ratios produces another equivalent ratio. Eventually, students should recognize, however, that the rows (or columns) of a ratio table are some (positive rational) multiple of each other, which will be important when they begin to compare ratios.

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## Part 1, Page 1.3

Focus: What patterns can you observe in a table of equivalent ratios?

Page 1.3 displays a ratio along with a group of circles and a group of squares that illustrate the ratio. The equivalent ratio is displayed in the table and represented by the corresponding shapes (squares and circles). Use the horizontal arrows at the


TI-Nspire
Technology Tips
The Reset button or ctrr dell returns the screen to the original setting The tab key on the handheld will toggle between the green and yellow arrows.

## Class Discussion

Help students to reason about the table of equivalent ratios by using both the visual images of the circles and squares and the numbers in the table.

Have students...
Set the original ratio to 2:3. Use the down arrow once and answer the following questions.

- Predict what you think will happen to the groups of circles and squares when you use the down arrow again.
- If you continue to click the down arrow, would you expect to see 8 green squares at some point? Explain your reasoning, and then check using the TNS activity.
- If there were 14 green squares, how many orange circles would you expect to see? Explain your thinking.
- Explain how 10 green squares for every 15 orange circles are related to 2 green squares for every 3 orange circles.

Look for/Listen for...

Possible answer: I would expect to get another set of 2 green squares and 3 orange circles.

Possible answer: I would expect to see 8 green squares because each time we are adding another set of 2 green and 3 orange, so the green squares would go from 4 to 6 to 8 .

Answer: I would expect to see 21 orange circles because for every 2 more green squares, the orange circles grow by 3 . And, for 12 green squares, there are 18 orange circles.

Possible answer: 10 green squares for 15 orange circles is shown as 5 rows each with 2 green squares and 3 orange circles.

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## Student Activity Questions

1. a. How is the ratio 3 green squares to 5 orange circles related to the ratio 18 green squares to 30 orange circles? Explain your reasoning.

Answer: They are equivalent ratios because the values in 18:30 are multiples of 3:5; 6(3):6(5).
b. Explain how the rows in the table generated for the ratio 3:5 are related.

Possible answer: All of the rows are equivalent to the ratio $3: 5$. Each row has a multiple of the original ratio, 3:5. Another way to think is additively; the first column in each row grows by 3 , and the second column in each row grows by 5 .

2 Reset the page and use the arrows at the top of the page to change the ratio to 5 green squares for every 2 orange circles. For each of the following, predict what you think the answer will be, and then check your answer using the TNS activity.
a. The number of orange circles for $\mathbf{2 0}$ green squares

Answer: 8
b. The number of green squares $\mathbf{1 2}$ orange circles

Answer: 30
c. Will you ever have $\mathbf{2 2}$ green squares? Why or why not?

Answer: No because the number of green squares has to be a multiple of 5 , and 22 is not a multiple of 5 . (There are no fractions of squares in the activity.)

> Teacher Tip: Remind students that to find equivalent ratios, they must multiply or divide both terms of the ratio by the same positive number.
d. How are the values in one row of the table related to the values in the next row?

Answers may vary. One possible answer is that each row can be found by adding 5 to the number of green way is to add 5 to the squares and 2 to the number of orange circles. A second way is to multiply the values in the ratio: 5:3 by consecutive positive whole numbers.

## Class Discussion

Have students... Look for/Listen for...

## Set the original ratio to 3 green squares per

 6 orange circles.- Predict what the $4^{\text {th }}$ row in the table will be. Possible answer: The $4^{\text {th }}$ row will have 12 green Use the TNS activity to check your prediction.
squares and 24 orange circles.


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## Class Discussion (continued)

- Which row of the table would contain 24 green squares? Explain how you found your answer.

Answer: The $8^{\text {th }}$ row of the table should have 24 green squares. Each row in the table is a multiple of the values in the ratio $3: 6$, and 3 times 8 would be 24.

Additional Discussion

## Have students...

Use the TNS activity to help explain reasoning.
As part of a rewards program, a restaurant gives out 4 reward stars for every 3 dollars spent.

- How many reward stars would you earn for spending $\$ 9.00$ ?
- How many reward stars would you earn if you spent \$1.50? Explain your reasoning.
- How could you use the TNS activity to help you find the number of dollars you would have to spend to earn 48 reward stars? Explain your reasoning.

Answer: 12 stars

Answer: 2 stars because 2:1.5 is equivalent to the ratio 4:3.

Possible answers:
(Strategy 1): Generate a table using the file up to 24 and 18 , then figure out from there by adding 4 s in the first column and 3 s in the second until you get to 48 in the first column, which would give you 48:36 so you would spend $\$ 36$ for 48 stars.
(Strategy 2): Move right from 24:18 by doubling to 48: 36 because doubling will give you an equivalent ratio.
(Strategy 3): Dividing 48 by 4 gives you 12, so finding an equivalent ratio by multiplying the values in the ratio $4: 3$ by 12 will give you 48:36.

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## Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Which of the following would be in the same table of equivalent ratios? $9: 6 ; 8: 5 ; 18: 12 ; 3: 2 ; 6: 4 ; 6: 9$. Explain how you found your answers.

Answer: The following ratios would be in the same table. They are equivalent to $3: 2$ because they are each "a positive number times 3:2": 9:6, 18:12, 6:4.
2. Fill in the blanks in each of the following to produce tables of equivalent ratios.
a.

| Lawns | Hours |
| :--- | :--- |
| 4 | 7 |
|  | 14 |
| 12 |  |
|  |  |

Answer:
a.

| Lawns | Hours |
| :--- | :--- |
| 4 | 7 |
| 8 | 14 |
| 12 | 21 |
| 40 | 70 |

b.

b.

c.

| Lawns | Hours |
| :--- | :--- |
| 1 |  |
| 2 |  |
| 3 | 12 |
|  | 16 |

c.

| Lawns | Hours |
| :--- | :--- |
| 1 | 4 |
| 2 | 8 |
| 3 | 12 |
| 4 | 16 |

3. If Sara used 3 cups of pancake mix to make 24 pancakes, how many cups would she need to make 40 pancakes?

Answer: 3:24 is equivalent to 1:8, which is equivalent to 5:40. So, the answer is 5 cups of pancake mix.

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## Student Activity Solutions

In this activity you will work together to explore equivalent ratios and examine patterns in tables of equivalent ratios. After completing the activity, discuss and/or present your findings to the rest of the class.

## Activity [Page 1.3]

1. a. How is the ratio 3 green squares to 5 orange circles related to the ratio 18 green squares to 30 orange circles? Explain your reasoning.

Answer: They are equivalent ratios because the values in 18:30 are multiples of 3:5; 6(3):6(5).
b. Explain how the rows in the table generated for the ratio $3: 5$ are related.

Possible answer: All of the rows are equivalent to the ratio 3:5. Each row has a multiple of the original ratio, 3:5. Another way to think is additively; the first column in each row grows by 3, and the second column in each row grows by 5.
2. Reset the page and use the arrows at the top of the page to change the ratio to 5 green squares for every 2 orange circles.

For each of the following, predict what you think the answer will be, and then check your answer using the TNS activity.
a. The number of orange circles for 20 green squares

Answer: 8
b. The number of green squares 12 orange circles

Answer: 30
c. Will you ever have 22 green squares? Why or why not?

Answer: No because the number of green squares has to be a multiple of 5 , and 22 is not a multiple of 5. (There are no fractions of squares in the activity.)
d. How are the values in one row of the table related to the values in the next row?

Answers may vary. One possible answer is that each row can be found by adding 5 to the number of green way is to add 5 to the squares and 2 to the number of orange circles. A second way is to multiply the values in the ratio: 5:3 by consecutive positive whole numbers.

