Building Concepts: Ratios and Fractions

Lesson Overview

This TI-Nspire™ lesson allows students to explore the differences and similarities between ratios and fractions. Students consider different ways to use ratios to describe a situation, thinking about the ratio of two different parts of a group or of a part of a group to the whole group. Students associate the “value” of a ratio with a fraction.

A ratio may be associated with a value; the *value* of a ratio $\boldsymbol{a}:\boldsymbol{b}$ is the quotient $\frac{a}{b}$ (if $\boldsymbol{b}$ is not 0).

Learning Goals

1. Identify the value associated with a ratio $a:b$ as the fraction $\frac{a}{b}$;
2. understand that value of the ratio $a:b$ compares a and b multiplicatively, indicating how many times as big a is as b.

Prerequisite Knowledge

Ratios and Fractions is the sixth lesson in a series of lessons that explore the concepts of ratios and proportional reasoning. This lesson builds on the concepts of the previous lesson. Prior to working on this lesson students should have completed *Ratio Tables* and *Comparing Ratios.* Students should understand:

- the concept of equivalent fractions;
- the concepts of ratios and equivalent ratios;
- how to complete ratio tables.

Vocabulary

- **rate**: the number of units of one quantity per the number of units of another quantity.
- **unit rate**: gives the number of units of one quantity per one unit of the other quantity.

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Ratios and Fractions_Student.pdf
- Ratios and Fractions_Student.doc
- Ratios and Fractions.tns
- Ratios and Fractions_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

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Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice, and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

This TI-Nspire™ lesson allows students to explore the differences and similarities between ratios and fractions. Ratios are different from fractions. A *ratio* is a pair of non-negative numbers, ***a*:*b*,** which are not both 0. As one of the numbers changes, the other changes by the corresponding multiplier. A ratio may be associated with a value; the *value* of a ratio ***a*:*b*** is the quotient $\frac{a}{b}$ (if ***b*** is not 0). The value of the ratio ***a:b*** compares **a** and **b** multiplicatively. If the ratio is ***a:b*,** then ***a*** is $\frac{a}{b}$ times as big as **b**. Specifically, $\frac{A}{B}$ is the factor that tells how many times as much of the first quantity *A* there is as of the second quantity *B*. (Similarly, the factor $\frac{B}{A}$ associated with the ratio *B*:*A* tells how many times as much of the first quantity in the ratio there is as of the second quantity.) For example, if there are three cups of peanuts in five cups of mixed nuts, the mixture is $\frac{3}{5}$ peanuts. If the mixture is 3 cups of peanuts to 2 cups of walnuts, then there are $\frac{2}{3}$ as many walnuts as peanuts.

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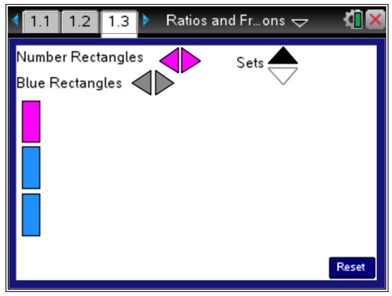
If a unit has been divided into five equal parts to obtain a unit fraction of $\frac{1}{5}$, the fraction $\frac{2}{5}$ is 2 copies of the unit fraction $\frac{1}{5}$. When that unit is divided into 3 times as many equal parts (15 where each part is $\frac{1}{15}$), then 3 times as many copies are needed to have the equivalent fraction $\frac{6}{15}$. If two quantities are in the ratio 2:5 and the first quantity is tripled to 6, then the second quantity must be tripled as well, to 15 which produces an equivalent ratio 6:15. Notice that each of the units that make up the fraction $\frac{6}{15}$ are smaller than each of the units that make up the fraction $\frac{2}{5}$. So while the ratio 6:15 involves 6 of the 15 parts and the ratio 2:5 involves 2 of the 5 parts, the ratios are equivalent.

The value of a ratio can be viewed as a unit rate in some contexts. A unit rate gives the number of units of one quantity per 1 unit of the other quantity. Multiplying by *N* will produce *N* units of the other quantity. Note that if you use the language of “part to whole” or “part to part” as the foundational language in talking about ratio, you will introduce a confounding factor when the idea of ratio is extended to rate of change, slope, and scaling.

Part 1, Page 1.3

Focus: How are ratios related to fractions?

Page 1.3 displays a column of pink and blue rectangles. The pink arrows set the number of rectangles in the column on the left. The arrows below the pink arrows set the number of blue rectangles in the set of rectangles. The vertical arrows generate multiples of the first column.



Students consider different ways to use ratios to describe a situation, basically thinking about the ratio of two different parts of a group or of a part of a group to the whole group. The language "part to part" and "part to whole" is deliberately avoided as this language is constraining and limits student thinking as they continue to work with ratios in situations such as those involving slope and similarity.

TI-Nspire Technology Tips

The tab key on the handheld will toggle between the arrows on the left.

The up/down arrow keys on the touchpad on the handheld will change the number of sets of rectangles shown.

Reset returns to the original screen or press ctrl del on handheld to reset.

Class Discussion

Have students…	**Look for/Listen for…**
Consider the blue and pink rectangles on page 1.3.	
• ***Write three different ratios represented in the display on the screen.***	Answer: 1 pink to 3 total rectangles (or 3 total: 1 pink); 2 blue rectangles to 3 total rectangles (or 3 total to 2 blue); 1 pink rectangle to 2 blue rectangles (or 2 blue: 1 pink).
• ***Change the ratio of pink to blue so there are 3 pink rectangles for every 1 blue rectangle. Describe the display***.	Answer: 4 rectangles with 3 pink and 1 blue**.**
• ***Find two ways to show the ratio 1 pink rectangle for every 2 blue rectangles. Describe the displays.***	Answer: There could be 3 rectangles, 1 pink and 2 blue, or there could be 6 rectangles with 2 pink and 4 blue.

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Class Discussion (continued)

Have students…	Look for/Listen for…
Set the ratio 2 pink to 4 blue rectangles. Use the vertical arrows to produce each of the following ratios. Explain how each diagram supports the concept of the ratio 2:4.	
• ***6 pink to 12 blue rectangles***	Answer: The 2:4 is repeated in each column; you have 3 columns each with 2 rows of pink and 4 rows of blue rectangles for a total of 6 pink and 12 blue.
• ***10 pink to 30 total rectangles***	Answer: 5 columns each with 2 rows of pink and 4 rows of blue rectangles for 10 pink and 20 blue rectangles, which is 10 pink to (10+20) total rectangles.
• ***1 set of pink to 2 sets of blue rectangles***	Answer: Using the original 2 pink to 4 blue, 1 set of pink would be 2 rectangles and 1 set of blue would be 2 blue rectangles. Thus, you would have 1 set of pink (2 pink rectangles in the set) to 2 sets of blue rectangles (4 blue rectangles in the 2 sets).

Student Activity Questions—Activity 1

1. Set the ratio to 3 blue rectangles out of a total of 5 rectangles. Estimate the missing value in each equivalent ratio, and then check your predictions using the TNS lesson. Explain why the answer makes sense in each case.

a. 9 blue out of ____ total number of rectangles

Answer: 9 blue out of 15 total rectangles. To have a ratio equivalent to 3:5 you would multiply each value by 3 to get 9:15.

b. 9 blue to ____ pink rectangles

Answer: 9 blue to 6 pink rectangles. If 3 out of the 5 rectangles are blue, the other 2 are pink, so the ratio of blue to pink is 3:2. An equivalent ratio would be to multiply both values by 3 to get 9:6.

c. ____ blue rectangles to 14 pink rectangles

Answer: 21 blue to 14 pink rectangles. The ratio of blue to pink is 3:2; so to have 14 pink, you would multiply each value in the ratio by 7 for a ratio of 21:14.

d. ____ pink rectangles out of 20 rectangles

Answer: 8 pink out of 20. The ratio of pink to the total number of rectangles is 2:5; so to have 20 total rectangles, you would multiply the values in the original ratio by 4 to get the equivalent ratio, 8:20.

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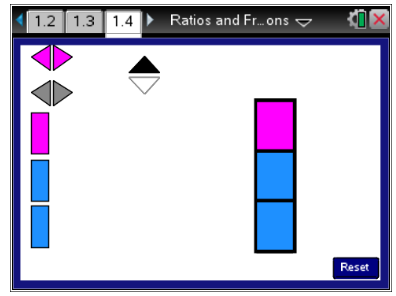
Part 2, Page 1.4

Focus: Students relate fractions to equivalent ratios.

This page works the same as page 1.3, except there is now a rectangle that is partitioned into according to the ratios set.



In the following questions, students associate the "value" of a ratio with a fraction. They investigate how a fraction bar displaying different unit fractions can be related to a collection of equivalent ratios and vice versa. The last problem connects the unit rate associated with a ratio to a fraction.

Class Discussion

- ***What is the ratio of blue to total rectangles on the left? What fraction of the bar on the right is blue?***

  Answer: The ratio is 2 blue to 3 total rectangles, and the fraction bar is $\frac{2}{3}$ blue.

- ***Select the up vertical arrow once. What happens to the ratio and what happens to the fraction bar?***

  Answer: There are 4 blue out of 6 rectangles, which is an equivalent ratio to 2:3 (each of the original number of blue and pink rectangles was doubled). The number of partitions in the fraction bar increased from 3 to 6, but the fraction bar did not change size.

A ratio a:b can be associated with a number, $\frac{a}{b}$, called its value.

- ***What is the value of the ratio 2:3?***

  Answer: the fraction $\frac{2}{3}$

- ***Do you think $\frac{2}{3}$ is a number? Why or why not?***

  Answer: Yes, $\frac{2}{3}$ is a number because it is 2 copies of the unit fraction $\frac{1}{3}$ and can be located on the number line at $\frac{2}{3}$ of a unit from 0.

- ***Reset the page. What is the ratio of the pink rectangles to the total number of rectangles? What is the value of that ratio?***

  Answer: The ratio of pink to total number of rectangles is 1:3, and its value would be $\frac{1}{3}$.

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Class Discussion (continued)

- ***Sam claims that the $\frac{1}{3}$ means that $\frac{1}{3}$ of all the rectangles on the left are pink. What would you say to Sam?***

  Answer: You are right. $\frac{1}{3}$ of the 3 rectangles would be 1 rectangle and only 1 is pink.

Use the vertical up arrow in the middle of the screen once.

- ***What is the ratio of pink to the total number of rectangles?***

  Answer 2:6

- ***What fraction is associated with the ratio 2:6?***

  Answer: $\frac{2}{6}$, which is equivalent to $\frac{1}{3}$.

- ***Make a prediction for what you think will happen if you use the vertical up arrow a second time. Use the TNS lesson to verify your prediction.***

  Answer: the ratio of pink to the total will be 3:9, and the fraction bar on the far right will show $\frac{3}{9}$ or 3 copies of the unit fraction $\frac{1}{9}$ as pink.

- ***What is happening to the number of the rectangles on the left and the number of partitions in the fraction bar on the right?***

  Answer: The number of pink and blue rectangles on the left is increasing and so is the number of partitions in the fraction bar.

- ***What is happening to the size of the rectangles on the left and the size of the unit fractions as you continue to select the up arrow?***

  Answer: The rectangles on the left stay the same size as the original rectangles; the unit fractions in the fraction bar are getting smaller.

Student Activity Questions—Activity 2

1. **Reset the page. Set the ratio for 2 blue to 3 pink rectangles. Which of the following statements are true? Explain your thinking.**

   a. **$\frac{2}{5}$ of the rectangles are blue rectangles.**

   Answer: True because $\frac{2}{5}$ of the 5 total rectangles is 2, the number of blue rectangles.

   b. **The number of blue rectangles is $\frac{2}{3}$ of the total number of rectangles.**

   Answer: False because $\frac{2}{3}$ of the 5 rectangles is not 2, the number of blue rectangles.

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Student Activity Questions—Activity 2 (continued)

c. **The number of blue rectangles is $\frac{2}{3}$ the number of pink rectangles.**

Answer: True because $\frac{2}{3}$ of the 3 pink rectangles is 2, the number of blue rectangles.

2. **For each of the following, predict what the fraction bar on the right would look like. You may want to use the TNS lesson to check your answers.**

a. **The ratio is 1 to 1.**

Answer: The fraction bar would be half pink and half blue, no matter what the unit fraction.

b. **The ratio is 1 pink rectangle for every 3 rectangles.**

Answer: The fraction bar would be $\frac{2}{3}$ blue and $\frac{1}{3}$ pink.

c. **The ratio is 12 pink rectangles to 8 blue rectangles.**

Answer: The fraction bar will be $\frac{12}{20}$ pink and $\frac{8}{20}$ blue, which is the same as $\frac{3}{5}$ pink and $\frac{2}{5}$ blue.

3. **What do each of the following tell you about the number of rectangles on the left?**

a. **The ratio of blue to pink rectangles is 15 to 5.**

Possible answer: There could be a total of 20 rectangles, 15 blue ones and 5 pink ones, but there could also be 40 rectangles with 30 blue ones and 10 pink ones.

b. **$\frac{1}{4}$ of the rectangles are pink.**

Answer: 1 of every 4 rectangles is pink, but you do not know how many rectangles there are.

c. **There are 3 times as many blue rectangles as pink ones.**

Answer: For every pink rectangle, there are 3 blue rectangles, but you do not know how many rectangles there are.

4. **A *rate* gives the number of units of one quantity per the number of units of another quantity. Find the rate of boys per girls in each case. You may want to use the TNS lesson to help your thinking.**

a. **$\frac{3}{5}$ of the class was boys.**

Answer: $\frac{3}{5}$ to $\frac{2}{5}$ or 3 boys per every 2 girls.

b. **Out of 40 students, 24 were boys.**

Answer: 24 boys to 16 girls or 3 boys per every 2 girls.

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Student Activity Questions—Activity 2 (continued)

c. **There were twice as many boys as girls.**

Answer: 2 boys per every one girl.

5. **Simon walks at a rate of 3 meters every 2 seconds. Which of the following describes his pace? You may want to use the TNS lesson to help your thinking.**

a. **1 meter every $\frac{2}{3}$ second**

b. **9 meters every 6 seconds**

c. **1.5 meters per second**

d. **45 meters every half a minute**

Answer: All of the choices are correct. The ratios in each case are equivalent to 3:2; for 1: $\frac{2}{3}$, multiply the values by 3; for 9:6, divide the values by 3; for 1.5:1, multiply the values by 2; for 45:30, divide the values by 15.

Additional Discussion

Reset and set the ratio to 3 blue rectangles and 1 pink rectangle. Use the vertical up arrow four times. Identify the following as True or False and explain your thinking in each case. In this situation, $\frac{3}{4}$ indicates...

- ***how many times as big the number of blue rectangles is to the total number of rectangles.***

  Answer: True because there are 20 rectangles and $\frac{3}{4}$ of 20 or 15 are blue.

- ***the total number of blue and pink rectangles is a multiple of 7***.

  Answer: False because the rectangles have to be in groups of 4, 3 blue and 1 pink, not groups of 7.

- ***the fraction of the total number of rectangles that is blue.***

  Answer: True because there are 20 rectangles all together and $\frac{3}{4}$ of 20 is 15, the number of blue rectangles; $\frac{15}{20} = \frac{3}{4}$ This is another way of saying the same thing as the first part.

Suppose the fractions $\frac{8}{24}$ blue and $\frac{16}{24}$ pink were represented in the fraction bar on page 1.4. Answer each of the following questions. Explain how the TNS lesson can help your thinking.

- ***Describe the ratio of the corresponding rectangles on the left.***

  Answer: Each unit fraction can be associated with one rectangle, so that means that 8 of the rectangles are one color (blue) and 16 the other color.

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Additional Discussion (continued)

- ***Sara says that $\frac{1}{3}$ of the rectangles are blue. Do you agree with Sara?***

  Possible answer: I agree because there are 24 rectangles, which can be visualized as 3 groups of 2 rows of 4 rectangles so there are 8 rectangles per group. 2 of the groups are pink and 1 group is blue, so 1 out of 3 of the groups is blue.

- ***Thad says there are 2 pink rectangles for every 1 blue rectangle. Do you agree with Thad?***

  Answer: Yes because two pink for every blue is equivalent to 16 pink to 8 blue, so the ratios are equivalent.

- ***Tina says there are $\frac{1}{2}$ as many blue rectangles as pink. Do you agree with Tina?***

  Answer: Tina is right because there are 16 pink rectangles and 8 blue rectangles so there are $\frac{1}{2}$ as many blue as pink rectangles.

- ***Suppose $\frac{1}{3}$ of the rectangles are blue. Would your thinking about the ratio of the corresponding rectangles be different than when starting with the fraction $\frac{8}{24}$? Why or why not?***

  Answer: The fraction $\frac{1}{3}$ could be associated with lots of different ratios so the rectangles might not be 8 blue and 16 pink. There could be 3 rectangles with 1 blue and 2 pink, or 6 rectangles with 2 blue and 4 pink, or lots of other equivalent ratios. However, if you started by reducing $\frac{8}{24}$ to $\frac{1}{3}$, your thinking might be the same.

Give at least two ratios that could be associated with each fraction. Sketch a picture of each.

- ***$\frac{1}{4}$ of the rectangles are pink.***

  Possible answer: 1 pink to 4 total rectangles; 2 pink to 8 rectangles.

- ***$\frac{6}{10}$ of the rectangles are pink.***

  Possible answer: 6 pink to 10 total rectangles; 3 pink to 5 total rectangles.

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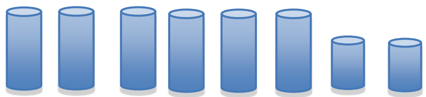
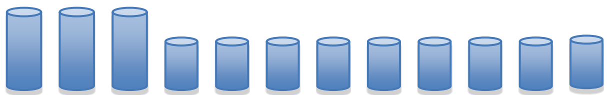
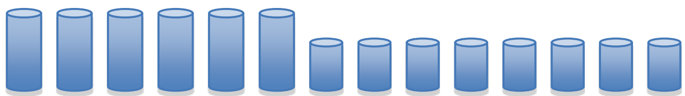
Sample Assessment Items

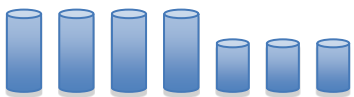
After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS lesson.

1. A cooler has $\frac{3}{4}$ as many large bottles of water as small ones. Which of the following could represent the number of bottles of water in the cooler?

   a. 

   b. 

   c. 

   d. 

   Answer: C

2. Steve made $\frac{3}{4}$ of his free throws. What is the ratio of the shots he made to the shots he missed?

   Answer: 3 to 1

3. The ratio of people over 45 years of age to those 45 years old and under in a certain region is 3:2. What fraction of the population in the region is 45 years or younger? ***Answer:*** $\boldsymbol{\frac{2}{5}}$

4. Lindsey found that 5 markers out of every batch of 30 were defective. What fraction of the markers was defective? ***Answer:*** $\boldsymbol{\frac{1}{6}}$

Student Activity Solutions

In these activities you will work together to use equivalent ratios and the relationship between ratios and fractions to solve problems. After completing each activity, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Set the ratio to 3 blue rectangles out of a total of 5 rectangles. Estimate the missing value in each equivalent ratio, and then check your predictions using the TNS lesson. Explain why the answer makes sense in each case.

 a. 9 blue out of ____ total number of rectangles

 Answer: 9 blue out of 15 total rectangles. To have a ratio equivalent to 3:5 you would multiply each value by 3 to get 9:15.

 b. 9 blue to ____ pink rectangles

 Answer: 9 blue to 6 pink rectangles. If 3 out of the 5 rectangles are blue, the other 2 are pink, so the ratio of blue to pink is 3:2. An equivalent ratio would be to multiply both values by 3 to get 9:6

 c. ____ blue rectangles to 14 pink rectangles

 Answer: 21 blue to 14 pink rectangles. The ratio of blue to pink is 3:2; so to have 14 pink, you would multiply each value in the ratio by 7 for a ratio of 21:14.

 d. ____ pink rectangles out of 20 rectangles

 Answer: 8 pink out of 20. The ratio of pink to the total number of rectangles is 2:5; so to have 20 total rectangles, you would multiply the values in the original ratio by 4 to get the equivalent ratio, 8:20.

Activity 2 [Page 1.4]

1. Reset the page. Set the ratio for 2 blue to 3 pink rectangles. Which of the following statements are true? Explain your thinking.

 a. $\frac{2}{5}$ of the rectangles are blue rectangles.

 Answer: True because $\frac{2}{5}$ of the 5 total rectangles is 2, the number of blue rectangles.

 b. The number of blue rectangles is $\frac{2}{3}$ of the total number of rectangles.

 Answer: False because $\frac{2}{3}$ of the 5 rectangles is not 2, the number of blue rectangles.

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c. The number of blue rectangles is $\frac{2}{3}$ the number of pink rectangles.

Answer: True because $\frac{2}{3}$ of the 3 pink rectangles is 2, the number of blue rectangles.

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Answer: 1 of every 4 rectangles is pink, but you do not know how many rectangles there are.

c. There are 3 times as many blue rectangles as pink ones.

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a. $\frac{3}{5}$ of the class was boys.

Answer: $\frac{3}{5}$ to $\frac{2}{5}$ or 3 boys per every 2 girls.

b. Out of 40 students, 24 were boys.

Answer: 24 boys to 16 girls or 3 boys per every 2 girls.

c. There were twice as many boys as girls.

Answer: 2 boys per every one girl.

5. Simon walks at a rate of 3 meters every 2 seconds. Which of the following describes his pace? You may want to use the TNS lesson to help your thinking.

   a. 1 meter every $\frac{2}{3}$ second

   b. 9 meters every 6 seconds

   c. 1.5 meters per second

   d. 45 meters every half a minute

   Answer: All of the choices are correct. The ratios in each case are equivalent to 3:2; for 1: $\frac{2}{3}$*, multiply the values by 3; for 9:6, divide the values by 3; for 1.5:1, multiply the values by 2; for 45:30, divide the values by 15.*

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