Results from the 2017 AP Calculus Administration AB-6

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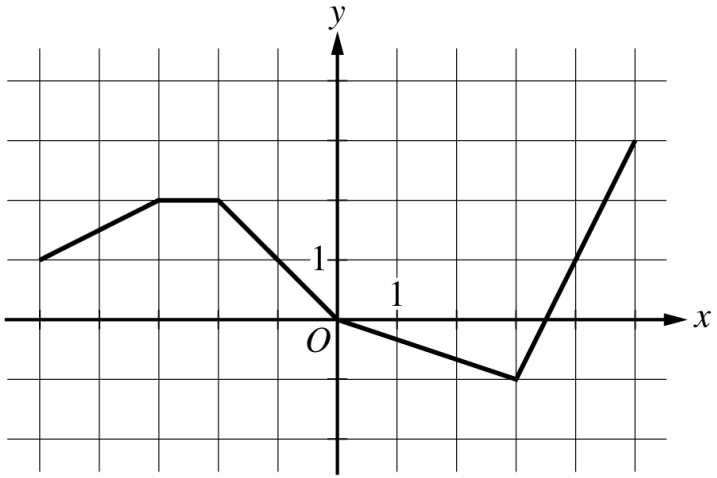
x	$g(x)$	$g'(x)$
–5	10	–3
–4	5	–1
–3	2	4
–2	3	1
–1	1	–2
0	0	–3



Graph of h

6. Let f be the function defined by $f(x) = \cos(2x) + e^{\sin x}$.

Let g be a differentiable function. The table above gives values of g and its derivative g' at selected values of x.

Let h be the function whose graph, consisting of five line segments, is shown in the figure above.

(a) Find the slope of the line tangent to the graph of f at $x = \pi$.

(b) Let k be the function defined by $k(x) = h(f(x))$. Find $k'(\pi)$.

(c) Let m be the function defined by $m(x) = g(-2x) \cdot h(x)$. Find $m'(2)$.

(d) Is there a number c in the closed interval $[-5, -3]$ such that $g'(c) = -4$? Justify your answer.

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(a) $f'(x) = -2\sin(2x) + \cos x\, e^{\sin x}$

$f'(\pi) = -2\sin(2\pi) + \cos \pi\, e^{\sin \pi} = -1$

$2 : f'(\pi)$

(b) $k'(x) = h'(f(x)) \cdot f'(x)$

$k'(\pi) = h'(f(\pi)) \cdot f'(\pi) = h'(2) \cdot (-1)$

$= \left(-\frac{1}{3}\right)(-1) = \frac{1}{3}$

$2 : \begin{cases} 1 : k'(x) \\ 1 : k'(\pi) \end{cases}$

(c) $m'(x) = -2g'(-2x) \cdot h(x) + g(-2x) \cdot h'(x)$

$m'(2) = -2g'(-4) \cdot h(2) + g(-4) \cdot h'(2)$

$= -2(-1)\left(-\frac{2}{3}\right) + 5\left(-\frac{1}{3}\right) = -3$

$3 : \begin{cases} 2 : m'(x) \\ 1 : m'(2) \end{cases}$

(d) g is differentiable. $\Rightarrow$ g is continuous on the interval $[-5, -3]$.

$\frac{g(-3) - g(-5)}{-3 - (-5)} = \frac{2 - 10}{2} = -4$

Therefore, by the Mean Value Theorem, there is a least one value c, $-5 < c < -3$, such that $g'(c) = -4$.

$2 : \begin{cases} 1 : \frac{g(-3) - g(-5)}{-3 - (-5)} \\ 1 : \text{justification,} \\ \quad \text{using Mean Value Theorem} \end{cases}$

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Student Performance

(1) Part (a): most made the connection between derivative and slope of the tangent line;

Chair Rule errors; sign errors, unit circle errors.

(2) Part (b): most made the connection between the derivative of h (presented graphically) with the slope of the line tangent ot the graph at the given point;

Chain Rule errors; arithmetic errors.

(3) Part (c): most were able to incorporate values from the table;

Product Rule or Chain Rule errors; $h'(2)$ errors; notational fluency.

(4) Part (d): justification of an incorrect answer (using IVT);

Errors in difference quotient;

Failure to confirm conditions for the Mean Value Theorem;

Insufficient verbal arguments.

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Part (a)

2: $f'(\pi)$

(1) Eligibility to earn any points:

Evidence of the Chain Rule and no Chain Rule errors.

$f'(\pi) = -1$ alone: 0/2

(2) To earn 2/2:

Communication of correct Chain Rule in both terms and $f'(\pi) = -1$.

- $f'(x) = -2\sin(2x) + (\cos x)e^{\sin x} = 1$
- $-2\sin(2\pi) + (\cos \pi)e^{\sin \pi}$
- $-2 \cdot 0 + (-1) \cdot e^0$

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Part (a)

2: $f'(\pi)$

To earn 1/2:

(1) Communication of correct Chain Rule in both terms and $f'(\pi) \neq -1$, or

(2) Partial communication of correct Chain Rule in both terms and $f'(\pi) = -1$.

(3) Examples

- $f'(\pi) = (\cos \pi)e^{\sin \pi}$
- $0 + (-1) \cdot e^0$

Notes

(1) Sign error in one term with no other errors: 1/2

(2) Two sign errors: 0/2

(3) Any Chain Rule error: 0/2

(4) Presentation errors.

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Part (b)

1: $k'(x)$

(1) Eligibility to earn any points:

Evidence of the Chain Rule and no Chain Rule errors.

$k'(\pi) = \dfrac{1}{3}$ alone: 0/2

(2) Examples:

- $h'(2) \cdot f'(\pi)$ 1 - ?
- $h'(f(\pi)) \cdot [f'(\pi)$ from part (a)$]$ 1 - ?
- $\left(-\dfrac{1}{3}\right)(-1)$ 0 - 0
- $k'(\pi) = \dfrac{1}{3}$ 0 - 0

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Part (b)

1: $k'(\pi)$

(1) Eligibility: evidence of the Chain Rule with no Chain Rule errors.

(2) Computation errors are assessed in the second point.

(3) $h'(f(\pi)) \cdot f'(\pi) = \dfrac{1}{3}$ 1 - 1

(Unless errors presented in intermediate work.)

(4) Import errors from part (a): $f'(\pi)$.

(5) Presentation errors:

$k'(\pi) = h'(f(\pi)) \cdot f'(\pi)$ alone 0 - 0

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Part (c)

2: $m'(x)$

Eligibility to earn any points: Product Rule structure: $g'(u)h(x) + h'(x)g(u)$

To earn 2/2:

(1) Communicates correct application of the Product Rule and the Chain Rule.

(2) $-2g'(-2x)h(x) + g(-2x)h'(x)$ 2 - ?

(3) $-2g'(-4)h(2) + g(-4)h'(2)$ 2 - ?

Note: $m'(2)$ point not earned yet. Must evaluate functions.

To earn 1/2:

(1) Chain Rule error and no other errors.

(2) $g'(-2x)h(x) + g(-2x)h'(x)$ 1/2 - ?

(3) $g'(-4)h(2) + g(-4)h'(2)$ 1/2 - ?

(4) $-2g'(-2)h(x) + g(-2x)h'(x)$ 1/2 - ?

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Part (c)

1: $m'(2)$

Eligibility:

(1) Earned at least 1 $m'(x)$ point.

(2) $g(u)$ and $g'(u)$ are in the table.

Examples

(1) $m'(2) = -2(-1)\left(-\frac{2}{3}\right) + (5)\left(-\frac{1}{3}\right)$ 1 - 1

(Partial communication of the Product Rule structure.)

(2) $m'(x) = -2g'(-2x) \cdot h'(x)$ 0 - 0

(3) $m(x) = -2g'(-2x) \cdot h(x)$ 0 - 0

(4) $m'(2) = -3$ alone 0 - 0

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Part (d)

1: $\dfrac{g(-3)-g(-5)}{-3-(-5)}$

A difference and a quotient using the endpoints of the interval $[-5, 3]$

Examples:

(1) $\dfrac{2-10}{-3-(-5)}$ 1 - ?

(2) $\dfrac{2-10}{2}$ or $\dfrac{-8}{-3-(-5)}$ 1 - ?

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Part (d)

1: $\dfrac{g(-3)-g(-5)}{-3-(-5)}$

Errors in evaluation:

$$\frac{2-10}{-3-(-5)} = -8$$ 1 - 0

(Unable to draw our conclusion.)

Presentation errors:

(1) $g(-3)-g(-5)/-3--5=-4$ 0 - ?

(2) $g(-3)-g(-5)/-3-(-5)=k\neq -4$ or alone 0 - 0

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Part (d)

1: justification, using Mean Value Theorem

Eligibility:

(1) Must confirm (in some way) the conditions for the MVT to apply: differentiable and continuous.

(2) Must draw the conclusion.

Examples (minimal):

(1) $g'(c) = \dfrac{g(-3) - g(-5)}{-3 - (-5)} = -4$

(Must include reference to $g'(c)$)

(2) $\dfrac{g(-3) - g(-5)}{-3 - (-5)} = -4$, so yes

(Without reference to c, must draw the conclusion explicitly.)