

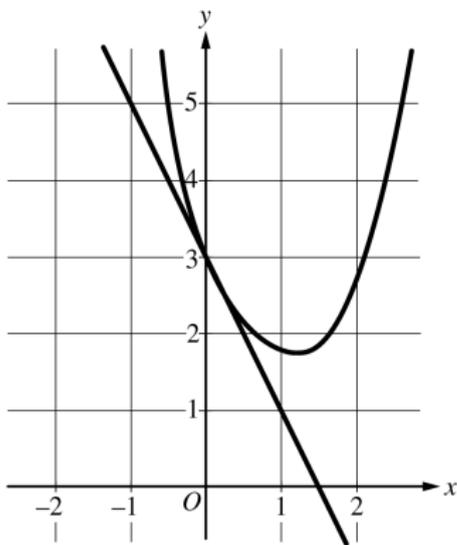
# TI in Focus: AP<sup>®</sup> Calculus

2019 AP<sup>®</sup> Calculus Exam: BC-6  
Scoring Guidelines

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## Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Student performance
- (4) Interpretation
- (5) Common errors
- (6) Specific scoring examples



$n$	$f^{(n)}(0)$
2	3
3	$-\frac{23}{2}$
4	54

6. A function  $f$  has derivatives of all orders for all real numbers  $x$ . A portion of the graph of  $f$  is shown above, along with the line tangent to the graph of  $f$  at  $x = 0$ . Selected derivatives of  $f$  at  $x = 0$  are given in the table above.

- (a) Write the third-degree Taylor polynomial for  $f$  about  $x = 0$ .
- (b) Write the first three nonzero terms of the Maclaurin series for  $e^x$ . Write the second-degree Taylor polynomial for  $e^x f(x)$  about  $x = 0$ .
- (c) Let  $h$  be the function defined by  $h(x) = \int_0^x f(t) dt$ . Use the Taylor polynomial found in part (a) to find an approximation for  $h(1)$ .
- (d) It is known that the Maclaurin series for  $h$  converges to  $h(x)$  for all real numbers  $x$ . It is also known that the individual terms of the series for  $h(1)$  alternate in sign and decrease in absolute value to 0. Use the alternating series error bound to show that the approximation found in part (c) differs from  $h(1)$  by at most 0.45.

(a)  $f(0) = 3$  and  $f'(0) = -2$

The third-degree Taylor polynomial for  $f$  about  $x = 0$  is

$$3 - 2x + \frac{3}{2!}x^2 + \frac{-23}{3!}x^3 = 3 - 2x + \frac{3}{2}x^2 - \frac{23}{12}x^3.$$

(b) The first three nonzero terms of the Maclaurin series for  $e^x$  are

$$1 + x + \frac{1}{2!}x^2.$$

The second-degree Taylor polynomial for  $e^x f(x)$  about  $x = 0$  is

$$\begin{aligned} 3\left(1 + x + \frac{1}{2!}x^2\right) - 2x(1 + x) + \frac{3}{2}x^2(1) \\ = 3 + (3 - 2)x + \left(\frac{3}{2} - 2 + \frac{3}{2}\right)x^2 \\ = 3 + x + x^2. \end{aligned}$$

$$2 : \begin{cases} 1 : \text{two terms} \\ 1 : \text{remaining terms} \end{cases}$$

$$2 : \begin{cases} 1 : \text{three terms for } e^x \\ 1 : \text{three terms for } e^x f(x) \end{cases}$$

$$\begin{aligned}
 \text{(c)} \quad h(1) &= \int_0^1 f(t) \, dt \\
 &\approx \int_0^1 \left( 3 - 2t + \frac{3}{2}t^2 - \frac{23}{12}t^3 \right) dt \\
 &= \left[ 3t - t^2 + \frac{1}{2}t^3 - \frac{23}{48}t^4 \right]_{t=0}^{t=1} \\
 &= 3 - 1 + \frac{1}{2} - \frac{23}{48} = \frac{97}{48}
 \end{aligned}$$

2 :  $\left\{ \begin{array}{l} 1 : \text{antiderivative} \\ 1 : \text{answer} \end{array} \right.$

(d) The alternating series error bound is the absolute value of the first omitted term of the series for  $h(1)$ .

$$\int_0^1 \left( \frac{54}{4!} t^4 \right) dt = \left[ \frac{9}{20} t^5 \right]_{t=0}^{t=1} = \frac{9}{20}$$

$$\text{Error} \leq \left| \frac{9}{20} \right| = 0.45$$

3 :  $\left\{ \begin{array}{l} 1 : \text{uses fourth-degree term} \\ \quad \text{of Maclaurin series for } f \\ 1 : \text{uses first omitted term} \\ \quad \text{of series for } h(1) \\ 1 : \text{error bound} \end{array} \right.$

## Student Performance

### Part (a)

- Most students were able to find  $f'(0)$  from the graph.  
Some students could not extract appropriate information from the graph.
- Good use of notation in presenting the components of the Taylor polynomial.
- Misinterpretation of *third-degree* as three non-zero terms or only the degree-three term.

### Part (b)

- Many students knew the first three non-zero terms for the Macluarin series for  $e^x$ .
- Some students presented an alternating series for  $e^x$  or omitted the constant term.
- To find the series for  $e^x f(x)$ , some students started to differentiate and evaluate at 0.
- Most common error: mistakes in multiplying two quadratic polynomials.
- Several students did not collect like terms to express their final answer as a Taylor polynomial.

## Student Performance

### Part (c)

- Most students were able to find an antiderivative of a polynomial and correctly evaluate the function on the interval  $[0, 1]$ .

- Most common error: misinterpretation of  $\int_0^x f(t) dt$

Read as an evaluation symbol.

### Part (d)

- Many students had trouble connecting the error bound, the value of the first omitted term, and 0.45.
- Some student tried to use the Lagrange Error Bound.
- Some students indicated that the actual error was 0.45.
- Many students used inconsistent or undefined notation.

**Part (a) 1: two terms; 1: remaining terms**

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Philosophy

- (1) We are looking for a correct degree 3 polynomial.
- (2) The first point is for two correct terms in the degree 3 polynomial.
- (3) The second point is for all remaining terms presented.

**Examples**

$$(1) 3 - 2x + \frac{3x^2}{2} - \frac{23x^3}{2 \cdot 3!} \qquad 1 - 1$$

$$(2) 3 - 2x + \frac{3x^2}{2} - \frac{23x^3}{12} \qquad 1 - 1$$

$$(3) f(x) = 3 - 2x + \frac{3x^2}{2} - \frac{23x^3}{12} \qquad 1 - 1$$

$$(4) 3 + 2x + \frac{3x^2}{2} - \frac{23x^3}{12} \qquad 1 - 0$$

**Part (a) 1: two terms; 1: remaining terms**

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Note: Any terms beyond the cubic term: student does not earn second point.

**Examples**

$$(1) 3 - 2x + \frac{3x^2}{2} - \frac{23x^3}{12} + \frac{54x^4}{4!} \qquad 1 - 0$$

$$(2) 3 - 2x + \frac{3x^2}{2} - \frac{23x^3}{12} + \dots \qquad 1 - 0$$

## Part (b) 1: three terms for $e^x$

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### Philosophy

- This point is earned for an explicit expression of the first three non-zero terms of the Maclaurin series for  $e^x$ .
- If more than three terms are listed, read the first three non-zero terms.
- The three terms may appear in a list.
- Use of sigma notation does not earn the point.
- An implicit expression of the first three non-zero terms of  $e^x$  as part of a product does not earn the first point.

**Part (b) Examples**

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$$(1) e^x = 1 + x + \frac{x^2}{2} \qquad 1 - ?$$

$$(2) 1 + x + \frac{x^2}{2} \qquad 1 - ?$$

$$(3) e^x = 1 + x + \frac{x^2}{2} + \dots \qquad 1 - ?$$

$$(4) e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} \qquad 1 - ?$$

$$(5) 1, x, \frac{x^2}{2} \qquad 1 - ?$$

$$(6) e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad ? - ?$$

$$(7) \left(1 + x + \frac{x^2}{2}\right) \left(3 - 2x + \frac{3x^2}{2}\right) \qquad ? - ?$$

## Part (b) 1: three terms for $e^x f(x)$

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- This point is earned for a correct second-degree Taylor polynomial.
- The product must be completed, and a second-degree polynomial must be presented.
- The terms must be collected.
- Special consideration for  $+\dots$  used in part (a) and part (b).
- Special case: First three non-zero terms of the Maclaurin series for  $e^x$  are incorrect.

**Part (b) Examples**

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$$(1) 3 + x + x^2 \qquad ? - 1$$

$$(2) e^x f(x) = 3 + x + x^2 \qquad ? - 1$$

$$(3) \left(1 + x + \frac{x^2}{2}\right) \left(3 - 2x + \frac{3x^2}{2}\right) \qquad 0 - 0$$

$$(4) 3 + (-2 + 3)x + \left(\frac{3}{2} - 2 + \frac{3}{2}\right)x^2 \qquad ? - 1$$

$$(5) 3 - 2x + \frac{3x^2}{2} + 3x - 2x^2 + \frac{3x^2}{2} \qquad ? - ?$$

**Part (c) 1: antiderivative**

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- Point is earned for a correct antiderivative.

$$\int_0^1 \left[ 3 - 2t + \frac{3}{2}t^2 - \frac{23}{12}t^3 \right] = 3t - t^2 + \frac{t^3}{2} - \frac{23}{48}t^4 \Big|_0^1 \quad 1 - ?$$

- The polynomial is written in part (a).  
An antiderivative may be presented without integral notation.

$$3t - t^2 + \frac{t^3}{2} - \frac{23}{48}t^4 \Big|_0^1 \quad 1 - ?$$

- If there is one error in the antiderivative, read for the answer point.

$$3x - x^2 + \frac{x^3}{3} - \frac{23}{48}x^4 \quad 0 - ?$$

- If there are two errors in the antiderivative, not eligible for the answer point.

$$3x - x^2 + \frac{x^3}{3} - \frac{23}{36}x^4 \quad 0 - 0$$

## Part (c) 1: answer

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- Point is earned for a correct answer; does not need to be simplified.
- If there is no integration, not eligible for the answer point.
- If simplification is attempted, it must be done correctly to earn the second point.
- Linkage errors in the antiderivative: do not earn the answer point.
- Possible to import a degree three polynomial if it has at least 2 non-zero terms.
- Special case: one error in the antiderivative.

**Part (d) 1: uses fourth-degree term of Maclaurin series for  $f$** 

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- Point is earned for correctly using a fourth-degree term of the series for  $f$ .

$$\int_0^x \frac{54t^4}{4!} dt \qquad 1 - ? - ?$$

- Explicit integral sign not necessary.

$$\frac{54x^5}{5 \cdot 4!}; \quad \frac{54x^5}{5!}; \quad \frac{54x^5}{120} \qquad 1 - ? - ?$$

- Error in the antiderivative, do not earn the second point.

$$\frac{54x^4}{5!}; \quad \frac{54x^5}{4!}; \quad \frac{54x^4}{120} \qquad 1 - 0 - ?$$

- The term may appear in a polynomial or a series.

$$3x - x^2 + \frac{x^3}{2} - \frac{23x^4}{48} + \frac{54x^5}{120} \qquad 1 - ? - ?$$

## Part (d) 1: uses first omitted term of series for $h(1)$

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- Need to see evidence the student is evaluating the correct fifth-degree term of  $h$ .
- Errors in antidifferentiation: do not earn the second point.
- The student must commit to this term.

$$\frac{54}{5!}; \quad \frac{54}{120}$$

? - 1 - ?

$$\left. \frac{54x^5}{5!} \right|_0^1$$

1 - 1 - ?

- The term cannot be part of a polynomial.

$$3x - x^2 + \frac{x^3}{2} - \frac{23}{48}x^4 + \frac{54x^5}{120} \left|_0^1\right.$$

1 - ? - ?

- Linkage error: do not earn the second point.

**Part (d) 1: error bound**

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- Must be an explicit reference to the error.  
error ;      verbal ;       $|h(1) - \text{value in (c)}|$
- Must make a connection to our stated value of 0.45

**Examples**

$$(1) \text{ error} \leq \frac{54(1)^5}{120} \leq 0.45 \qquad ? - 1 - 1$$

$$(2) \left| h(1) - \frac{97}{48} \right| \leq \frac{54(1)^5}{5!} \leq 0.45 \qquad ? - 1 - 1$$

$$(3) \frac{54(1)^5}{5!} \leq 0.45 \qquad ? - 1 - 0$$

$$(4) \text{ error} \leq \frac{54(1)^5}{5!} = \frac{54}{120} \qquad ? - 1 - 0$$

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