TI in Focus: AP® Calculus

2021 AP® Calculus Exam: BC-2

Technology Solutions and Problem Extensions

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Outline

(1) Free Response Question

(2) Scoring Guidelines

(3) Solutions in greater detail

(4) Solutions using technology

(5) Problem Extensions

AP® Calculus BC 2021 Free-Response Questions

2. For time $t \geq 0$, a particle moves in the xy-plane with position $(x(t), y(t))$ and velocity vector $\left\langle (t-1)e^{t^2}, \sin\left(t^{1.25}\right) \right\rangle$. At time $t = 0$, the position of the particle is $(-2, 5)$.

   (a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.

   (b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 1.2$.

   (c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

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(a) Find the speed of the particle at time $t = 1.2$. Find the acceleration vector of the particle at time $t = 1.2$.

$\sqrt{(x'(1.2))^2 + (y'(1.2))^2} = 1.271488$ At time $t = 1.2$, the speed of the particle is 1.271.	Speed	**1 point**
$\langle x''(1.2), y''(1.2) \rangle = \langle 6.246630, 0.405125 \rangle$ At time $t = 1.2$, the acceleration vector of the particle is $\langle 6.247 \text{ (or } 6.246), 0.405 \rangle$.	Acceleration vector	**1 point**

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(b) Find the total distance traveled by the particle over the time interval $0 \le t \le 1.2$.

$\int_0^{1.2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 1.009817$	Integrand	**1 point**
The total distance traveled by the particle over the time interval $0 \le t \le 1.2$ is 1.010 (or 1.009).	Answer	**1 point**

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(c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

$x'(t) = (t-1)e^{t^2} = 0 \Rightarrow t = 1$	Sets $x'(t) = 0$	**1 point**
Because $x'(t) < 0$ for $0 < t < 1$ and $x'(t) > 0$ for $t > 1$, the particle is farthest to the left at time $t = 1$.	Explains leftmost position at $t = 1$	**1 point**
$x(1) = -2 + \int_0^1 x'(t)\,dt = -2.603511$ $y(1) = 5 + \int_0^1 y'(t)\,dt = 5.410486$	One coordinate of leftmost position	**1 point**
The particle is farthest to the left at point $(-2.604 \text{ (or } -2.603), 5.410)$.	Leftmost position	**1 point**
Because $x'(t) > 0$ for $t > 1$, the particle moves right for $t > 1$. Also, $x(2) = -2 + \int_0^2 x'(t)\,dt > -2 = x(0)$, so the particle's motion extends to the right of its initial position after time $t = 1$. Therefore, there is no point at which the particle is farthest to the right.	Explanation	**1 point**

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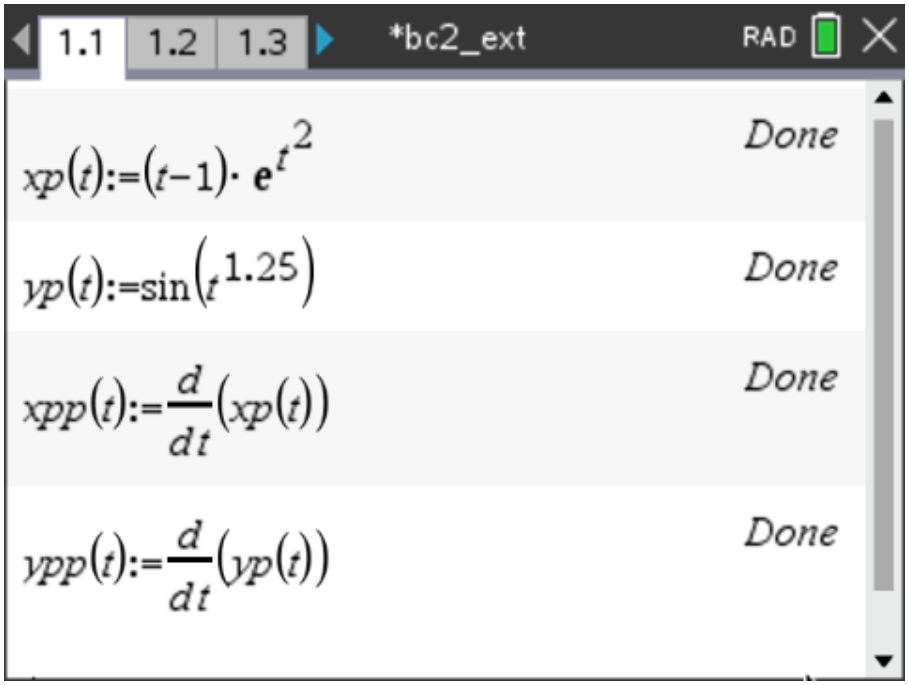
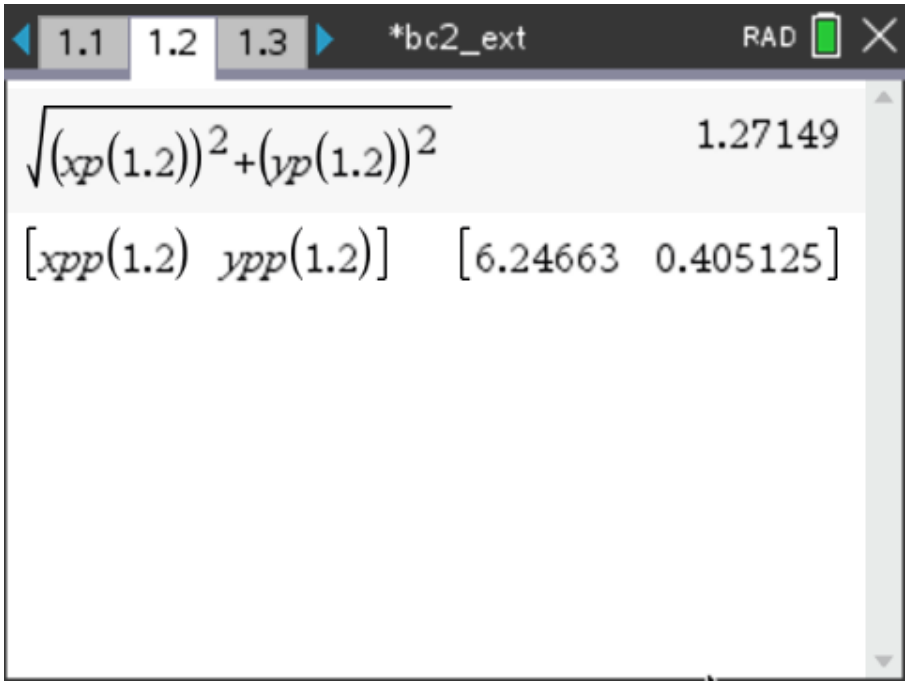
Part (a)

speed $= \sqrt{[x'(1.2)]^2 + [y'(1.2)]^2} = 1.271$

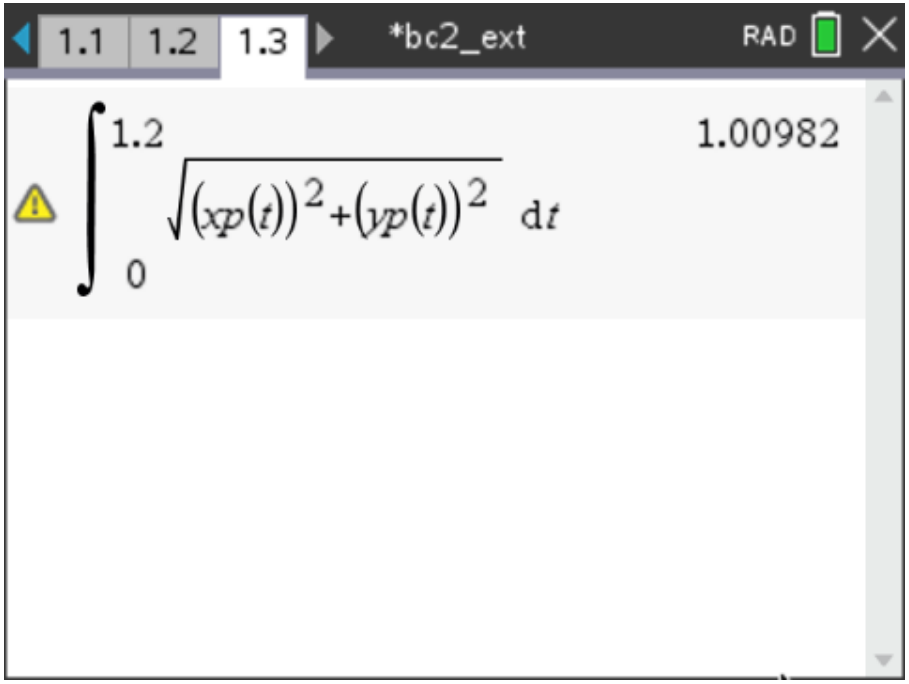
acc vector $= \langle x''(1.2), y''(1.2) \rangle = \langle 6.247, 0.405 \rangle$





Part (b)

$$D = \int_0^{1.2} \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt = 1.010$$



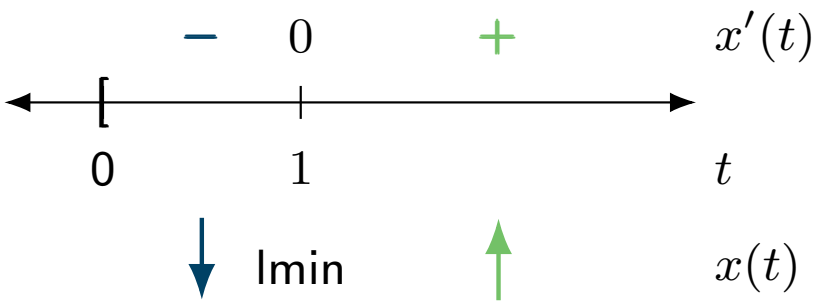
The total distance traveled by the particle over the time interval $0 \le t \le 1.2$ is 1.010.

Part (c)

(c) Find the coordinates of the point at which the particle is farthest to the left for $t \geq 0$. Explain why there is no point at which the particle is farthest to the right for $t \geq 0$.

$x'(t) = (t-1)e^{t^2} = 0$ Find critical points

$t - 1 = 0 \quad \Rightarrow \quad t = 1 \qquad e^{t^2} \neq 0$ Principle of Zero Products



For $0 \leq t < 1$, $x'(t) < 0$ and for $t > 1$, $x'(t) > 0$.

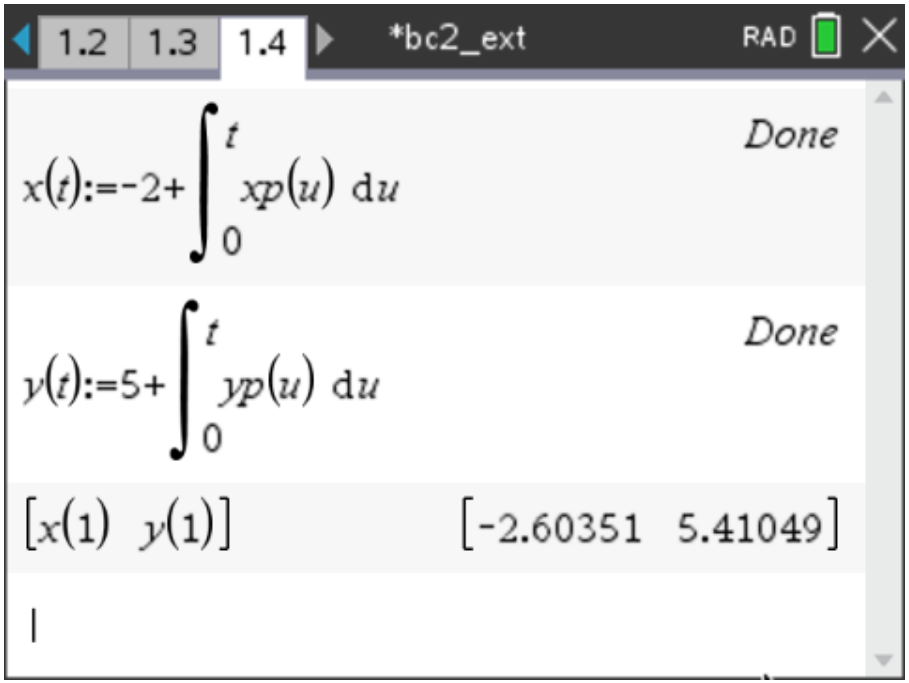
Therefore, the particle is farthest to the left at time $t = 1$.

Part (c)

Use the Net Change Theorem (twice) to find the position of the particle at time $t = 1$.

$$x(1) = -2 + \int_0^1 x'(t)\,dt = -2.604$$

$$y(1) = 5 + \int_0^1 y'(t)\,dt = 5.410$$



Part (c)

$x'(t) > 0$ for $t > 1$. Therefore, the particle moves right for $t > 1$

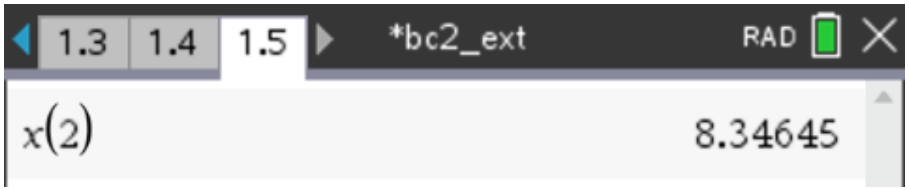
Farthest point to the right *could* be the position of the particle at time $t = 0$: $(-2, 5)$

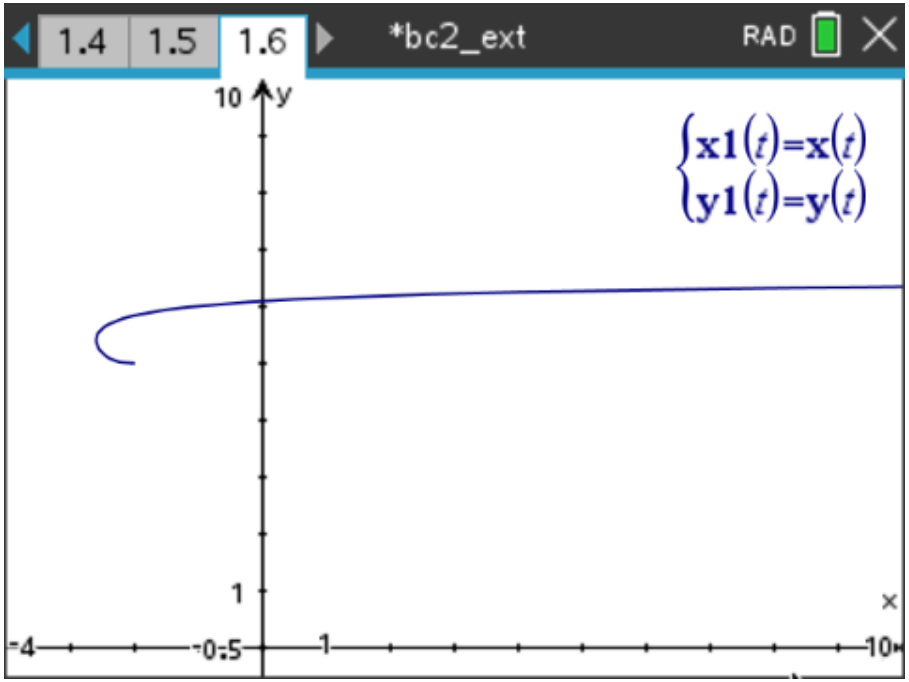
$$x(2) = -2 + \int_0^2 x'(t)\,dt = 8.346 > -2$$

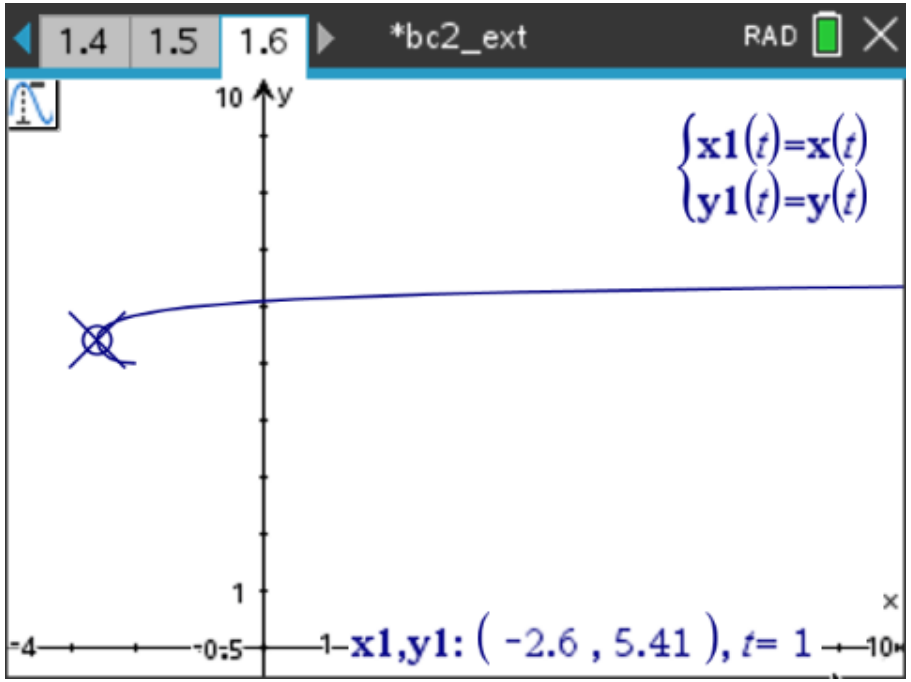
The particle's motion extends to the right of its initial position after time $t = 1$.

There is no point at which the particle is farthest to the right.

Part (c)


1.3 1.4 1.5
*bc2_ext
RAD
x(2)
8.34645


1.4 1.5 1.6
*bc2_ext
RAD
10
y
x1(t)=x(t)
y1(t)=y(t)
1
x
-4
-0.5
1
10


1.4 1.5 1.6
*bc2_ext
RAD
10
y
x1(t)=x(t)
y1(t)=y(t)
1
x
-4
-0.5
1
x1,y1: (-2.6 , 5.41), t= 1
10

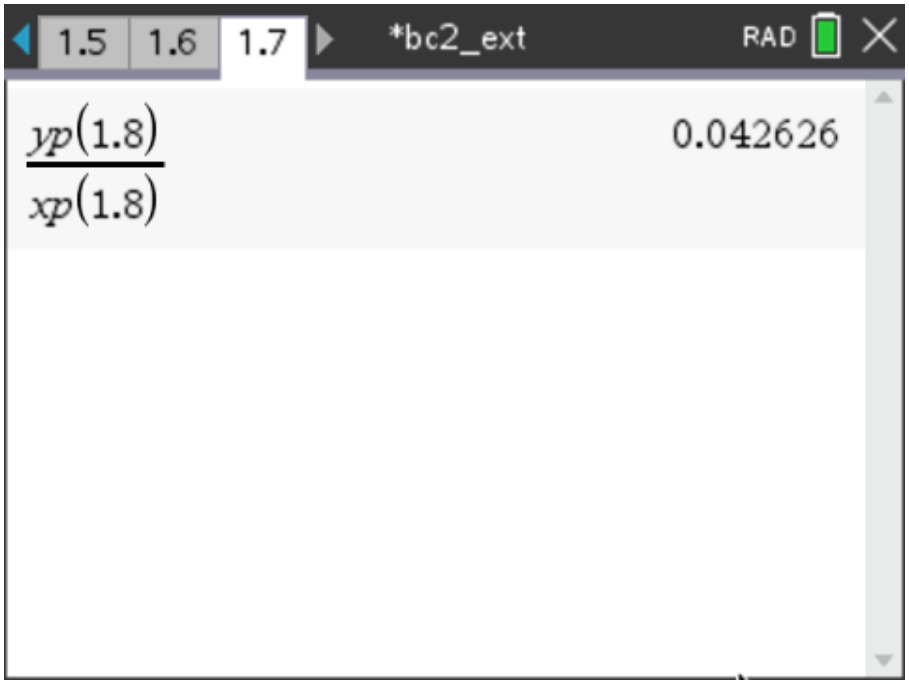
(d) Find the slope of the tangent line to the curve at time $t = 1.8$.

Solution

The slope of the tangent line is the ratio of the vertical and horizontal velocities.

Find $\dfrac{dy}{dx} = \dfrac{y'(t)}{x'(t)}$ at $t = 1.8$

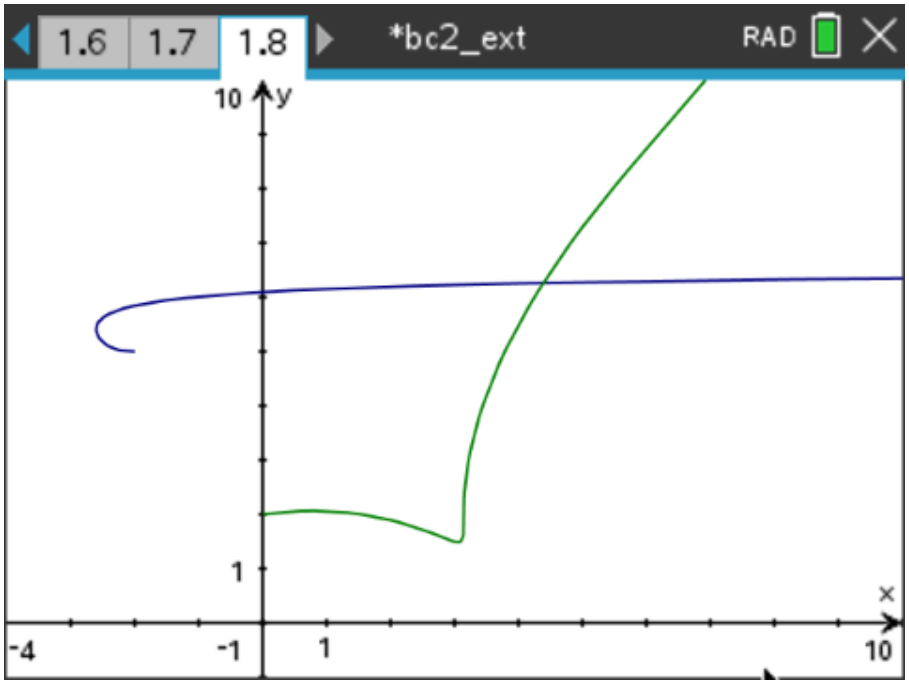
$$\left.\frac{dy}{dx}\right|_{t=1.8} = \frac{y'(1.8)}{x'(1.8)} = 0.043$$



Sketch the velocity and acceleration vectors.

(e) For time $t \geq 0$, a second particle moves in the xy-plane with position $(x_2(t), y_2(t))$ such that $x_2(t) = t + \sin t$ and $y_2(t) = e^{t/3} + \cos t$. Are the particles ever at the same position at the same time?

Solution

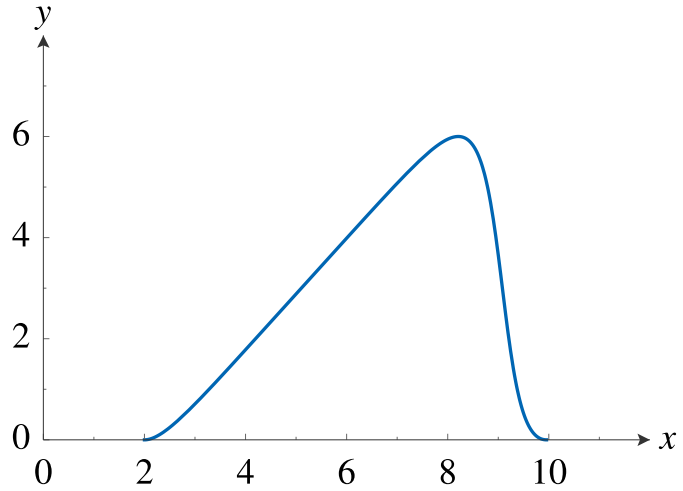


Example

A particle moving along a curve C in the xy-plane is at position $(x(t), y(t))$ for $0 \le t \le 2\pi$, where

$$\frac{dx}{dt} = e^{\sin t} \qquad y(t) = 3(1 - \cos t)$$

and $x(t)$ is not explicitly given. At time $t = 0$ the particle is at the point $(2, 0)$, and the curve C is shown in the figure.



(a) Find the velocity vector at time $t = \pi/4$. Find the acceleration vector at time $t = \pi/4$.

(b) Find the first time t in the interval $0 \le t \le 2\pi$ for which the speed of the particle is 3.

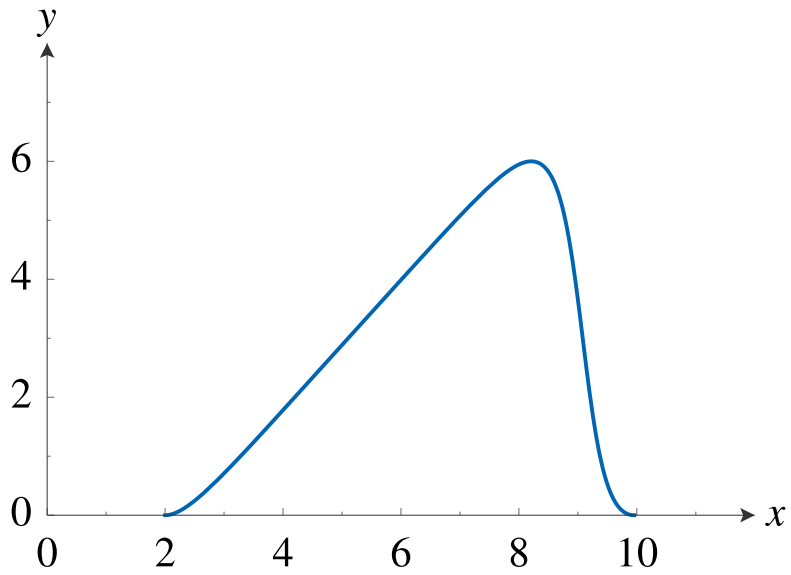
Example 1 (Continued)

(c) Find the coordinates of the highest point on the curve C for $0 \leq t \leq 2\pi$.

(d) Find the position of the particle at time $t = \pi/4$.

(e) Find the total distance traveled by the particle for $0 \leq t \leq 2\pi$.

(f) Find the point on the curve C where the slope of the tangent line is -2.



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