# The Pentagon Problem

TEACHER NOTES

ENHANCED FOR THE ESSENTIAL MATH TEACHING PRACTICES

*These Teacher Notes were enhanced to demonstrate how to integrate the Eight Essential Mathematics Teaching Practices into a lesson, with specific examples for when and how to address students' questions and work in class. Look for the Math Teaching Practices, which have been noted in italics, throughout the lesson to encourage student thinking and create effective mathematical discourse.*

## About the Mathematics

The Pentagon Problem asks students to create a new pentagon from a given one, keeping the area the same. The document, *The_Pentagon_ Problem.tns* was created in response to poor student performance on a 2005 National Assessment of Educational Progress (NAEP) eighth grade test item.

The document provides a simple, virtual "geoboard" that allows students to manipulate the polygon displayed on a unit grid. The grid squares provide a visual means of finding area.

With the .tns file, students can use the concrete picture to help solve the problem and to check their answers to the problem using different methods. They can also use the document to justify conclusions about area, to communicate conclusions to others and to distinguish correct reasoning from that which is flawed.

***Implement tasks that promote reasoning and problem solving:*** *This lesson places higher-level demands on students, as they must connect the meaning of area to various, free-form polygons. Students learn to visualize the composite area as the sum of the areas of known shapes. The deductive reasoning needed for this activity helps students to build connections to meaning in measurement and definition of polygons. Students should be encouraged to demonstrate more than one way to solve the problem and to share their strategies. They might be asked to identify which strategies are correct and which are not and to explain their reasoning.*

*Allowing students the time to visualize, think about and discuss the concepts in this activity helps them to develop a deeper meaning of the mathematics. Students may work independently or in small groups for this lesson. It is important for the teacher to monitor students' work, to ask questions throughout the lesson and to include class discussion in order to ensure student understanding.*

**Principles to Actions:**

This lesson includes a guide to using the Essential Mathematics Teaching Practices, as described in *Principles to Actions: Mathematics Success for All.*

**Tech Tips:**

- This activity includes screen captures taken from the TI-Nspire™ CX handheld. It is also appropriate for use with the TI-Nspire™ family of products including TI-Nspire™ software and TI-Nspire™ App for iPad®. Slight variations to these directions given within may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

**Lesson Files:**

- The_Pentagon_Problem.tns

## Math Objectives and Learning Goals

***Establish mathematics learning goals to focus learning.*** *Prior to doing this lesson with students, you should establish the math objectives – which state what students will do as a part of the lesson – and the learning goals – that state what the students will understand as part of the lesson. Knowing those goals and objectives will help you focus your questions and guide discussions with students.*

- ***Learning Goal:*** *Students will understand that the area for a composite shape can be found by decomposing the shape and finding the sum of the areas of each of the composing shapes.*
- **Math Objective:** Students will find several non-congruent pentagons having the same area as a given pentagon.
- ***Learning Goal:*** *Students realize that the area of triangles and parallelograms remains the same as long as the length of the base and height remain the same.*
- **Math Objective:** Students will verify that areas of concave and convex pentagons are the same.
- **Math Objective:** Students will solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms (CCSS Geometry, Grade 7).
- **CCSS Mathematical Practices:**
    - Students will make sense of problems and persevere in solving them (CCSS MP1).
    - Students will construct viable arguments and critique the reasoning of others (CCSS MP3).
    - Students will reason abstractly and quantitatively (CCSS MP2).
    - Students will look for and make use of structure (CCSS MP7).
    - Students will look for and express regularity in repeated reasoning (CCSS MP8).

## Activity Materials

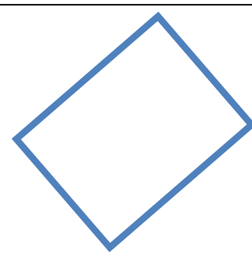
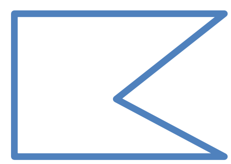
Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

## TI-Nspire™ Navigator™ System

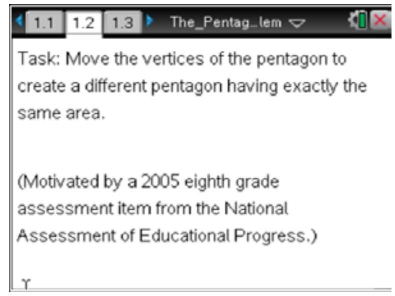
- Send and collect a file.
- Use Class Capture to monitor student work.
- Use Live Presenter to let students demonstrate their work.

***Support productive struggle in learning mathematics.*** *This activity challenges students to think differently about their approach to finding area of shapes. The ability to drag points to a possible solution, check the solution, then try another approach provides students with opportunities to struggle with the task in an environment that can help them think about the mathematical properties of the area of shapes.*

***Elicit and use evidence of student thinking:*** *Most likely, students are familiar with finding area analytically for a fixed set of shapes: triangle, rectangle or circle. They may have some basic understanding of finding the area of composite shapes. Start the lesson by asking students about various shapes that are drawn on the board; starting with the shape on the left and working to the right. Allow the students to share their thinking with the class as they answer the following. It is important for you to allow students time to internalize and respond to each other.*

| | | |
|---|---|---|
|  |  |  |
| *Is this a quadrilateral? How do you know?*<br><br>*How would you find the area of this shape?* | *Is this a quadrilateral? How do you know?*<br><br>*How would you find the area of this shape?*<br><br>*Note: This is a good place for students to discuss the idea of enclosing a shape with a rectangle and subtracting areas between it and the shape to find the area of the interior, irregular shape.* | *Is this a quadrilateral? How do you know?*<br><br>*How would you find the area of this shape?* |

## Using the Document




Page 1.2 describes the task: create a different (meaning not congruent) pentagon containing the same area as the given pentagon.

**Tech Tip:** To move a point, press on it once. This brings up a menu of options. Select the item you want to move and press Done. The point is now ready to move.

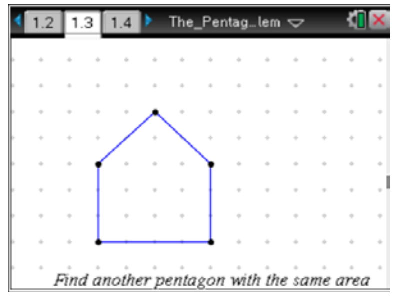
***Facilitate meaningful mathematical discourse.*** *It is important to allow your students to discuss this problem in order to give them an opportunity to analyze and compare the ideas of others. As you listen to students work together, think about how you want to sequence students' answers for explanation to the whole class. The order in which these questions are posed is important to helping students build their understanding of the concepts. Be sure that students have time to internalize each response before moving on to the next point.*

| ***Practices for effective discourse*** | ***Teacher response/ actions*** |
|---|---|
| *Anticipating student responses* | *Prior to the lesson, think about the types of answers your students may give and how you may respond. It is easier to approach the lesson with a plan for the discussion than attempting to do it on the spot.* |
| *Monitoring student work* | *As students work through the problem, make some observations about student work. Rather than providing a direct path to an answer, allow students time to reason on their own or to discuss possibilities with another student. If a student is stuck, you may ask questions to help them get started.* |
| *Selecting students to present work* | *As you observe students' work, identify solutions that can be presented to the class or group. Ask students to present their ideas in the following order.* |
| *Sequencing students' responses* | *Select student solutions to help build ideas:* |

| | |
|---|---|
| | *1. From the students' work, identify 2 different pentagons (that look different from the original) and ask students to discuss what makes each a pentagon.*<br>*2. Students may have different methods for finding the area of the pentagon. Ask students to share their reasoning in the following order:*<br>*a. Area found by counting or calculating area of smaller shape;*<br>*b. Area is found using formulas such as base x height to find areas of the smaller shapes that compose the pentagon;*<br>*c. Area is found by using subtraction by enclosing a shape in a larger simple shape and subtracting the excess area;*<br>*d. Area is found by using a compensation strategy: adding area on one side and subtracting it on another;*<br>*e. Area is found by using the common structure of height and base of parallelograms or triangles as they change the shape of the original pentagon.*<br>*The first two methods are common, but it is important for students to see other methods. If your students do not provide alternatives like those listed here, ask them to think about other ways that they could find area other than those shown already. This should encourage students to think about solving the problem in a different way.* |
| *Connecting students' responses to key ideas* | *What rules do shapes need to follow in order to be called pentagons? Use the file to give examples of shapes that are not pentagons.*<br><br>*How do you know that the areas are equal?*<br><br>*Are any of the pentagons that we explored here congruent?* |

| | |
|---|---|
| | *How do you know?* |

Page 1.3 provides a dynamic picture of a pentagon on a square grid. Students can move the vertices to create a new polygon. The grid squares become useful in determining the area of the polygon.

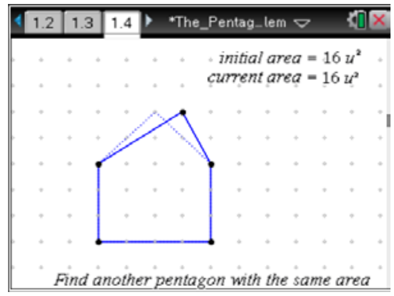



***Pose purposeful questions:*** *Follow the students' discussions from the previous problem with questions that help solidify the concepts. Be sure to ask questions as they relate to student work. Be mindful of the questions you ask so that they do not lead directly to an answer. Allow students time to think and reason. Their responses will help identify the route you should take with the next question.*

| ***Teacher Question*** | ***Question Type*** |
|---|---|
| *How do you know if a shape is a pentagon?* | *Gathering information* |
| *If you have an irregular shape cut out of paper, how could you measure the area?* | *Making the mathematics visible* |
| *What are different ways to verify that a solution is correct?* | *Making the mathematics visible* |
| *Explain the meaning of composite figures and how you can find the area of such shapes.* | *Making the mathematics visible* |
| *Are shapes that have the same area congruent? How do you know?* | *Encouraging reflection and justification* |
| *Do shapes that are congruent have the same area? How do you know?* | *Encouraging reflection and justification* |
| *Can you come up with a solution that moves only one vertex? Only two vertices? Why does that solution work?* | *Encouraging reflection and justification* |
| *What are the advantages and disadvantages of different solutions?* | *Encouraging reflection and justification* |

***Use and connect mathematical representations.*** *Students can make connections among the area formulas for triangles, rectangles and parallelograms. By connecting the symbolic formulas for area to the graphical representations of the components of those formulas, in particular identifying the height in the graph of a triangle or quadrilateral, students can explore why the formulas make sense and when they can be used.*

Page 1.4 adds area calculations for checking student solutions.




## Discussion Possibilities

There are many possible correct answers (and many possible incorrect ones as well). A mathematically-productive classroom discussion can be centered on considering various student solutions, strategies, and approaches.

**TI-Nspire™ Navigator™ Opportunity: *Class Capture and Live Presenter***

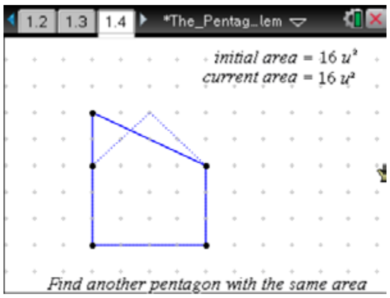
**See Notes 1 and 2 at the end of this lesson.**

## Examples

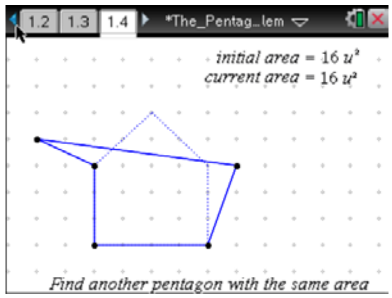
Here are six possible student solutions (A-F). Which are correct? Which are incorrect? And why?

Spur discussion by asking individual students to elaborate on the strategy each used to find a solution (use of a formula? A “compensation” method—i.e., imagining subtracting part of the original and “pasting” it into a new position?).

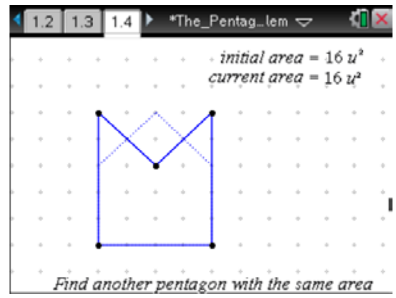
**A.**



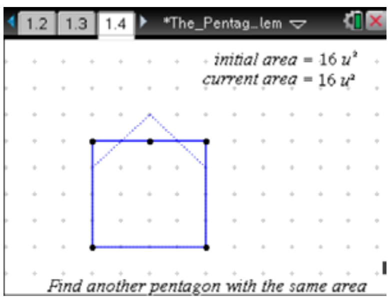

**B.**



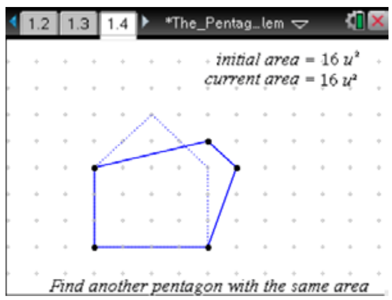

**C.**



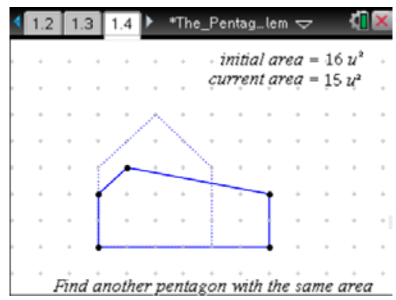

**D.**




**E.**




**F.**




## Answers

A. **Incorrect**: It is not a pentagon, although it has the correct area.

B. **Correct**: Students might have used a compensation strategy where the rectangular part remains constant and the triangular section has been divided in half. They then place one triangle on the upper left and the other triangle off the right side of the rectangle to create a new pentagon.

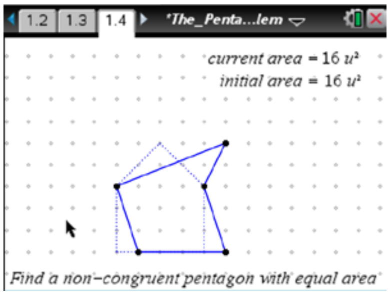
C. **Correct**: Although it is concave, it is still a pentagon.

D. **Incorrect**: It is not a pentagon. Some might argue that the presence of the point indicates is it made up of five segments and thus should be considered a pentagon.

E. **Correct**: One way of looking at it is Area=15-1.5+2.5=16. Rectangle 3x5 minus the triangle at the right side plus the area of the top triangle.

F. **Incorrect**: Area is 15. (rectangle 6x2) + (triangle (1/2 of 6x1) = 15.

The example to the right can be shared with the class. Ask students to discuss strategies for finding the area of this shape. This problem may be helpful for students who struggled at the beginning of the lesson in identifying a procedure for finding the area of a slanted parallelogram.




## TI-Nspire™ Navigator™ System

**Note 1 Class Capture**

Use Class Capture to share student solutions anonymously and to afford students the opportunity to critique and evaluate each other's reasoning.

The possible student solutions above might be typical of the ones that would be on a Class Capture of student work. Discussion should focus on how students might have strategized and whether the solutions are in fact correct.

**Note 2 Live Presenter**

Using *Live Presenter* gives a student control over TI-Nspire™ Navigator™ system's display to show how they moved vertices in creating a new pentagon and to explain their reasoning for doing so.