35 YEARS OF IMPAC(T³™)

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Teachers Teaching with Technology™

International Conference

AP® Calculus:
Playing the Averages -
Leveraging the Mean Value Theorems to Build Student Understanding

Presenter

Kim Thomas

T^3™ National Instructor

Peoria, Arizona

@Kim_Math

Presenter

Dennis Wilson

T^3™ National Instructor

Fairburn, Georgia

@EyesOfNewton

Agenda

- Define Average
- Explore Average Rate of Change
- Investigate the Mean Value Theorem
- Connect with the Mean Value Theorem for Integrals

What is Average?

Paige has 1 piece of bubble gum and Tami has 2 pieces. Deji decides to share his 6 pieces with Paige and Tami so all three can have the same amount of bubble gum. Determine how many pieces of bubble gum each person will have after Deji shares his pieces with his friends.

Mean Value Theorem

If a function f is continuous over the closed interval $[a,b]$ and differentiable over the open interval (a,b), then there exists some c, $a < c < b$, such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Mean Value Theorem

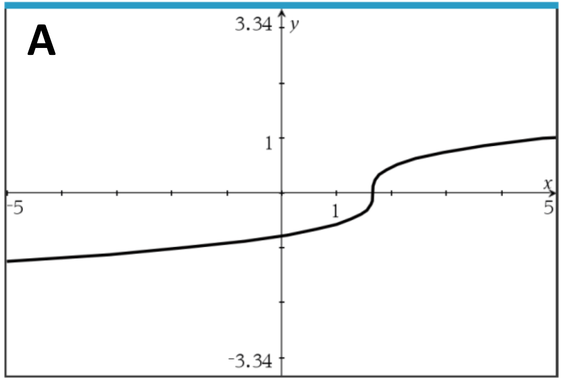
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

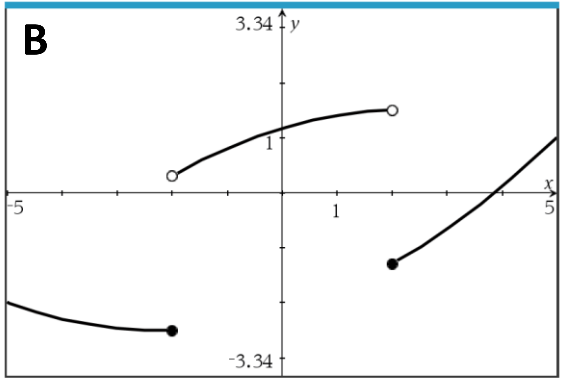
Instantaneous Rate of Change = Average Rate of Change

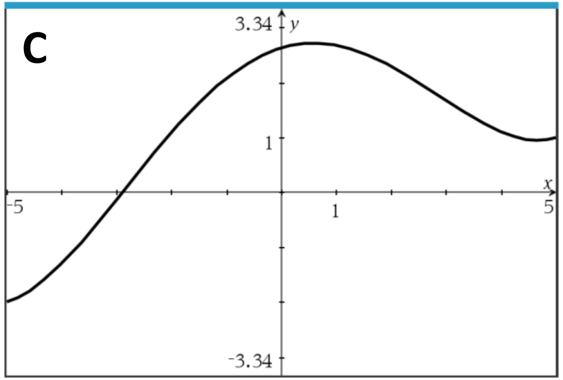
Tangent Line is parallel to the Secant Line

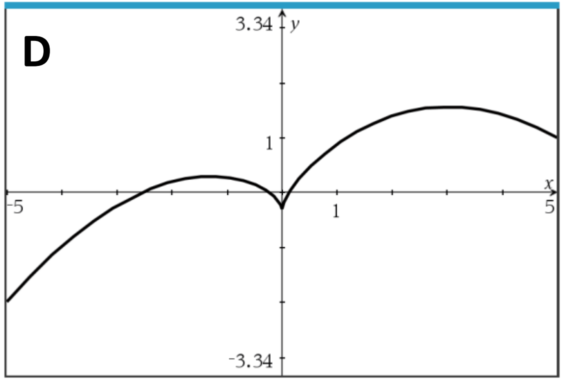
Multiple Choice

The Mean Value Theorem can be applied to which of the given functions on the closed interval [-5,5] as shown in each of the given graphs?









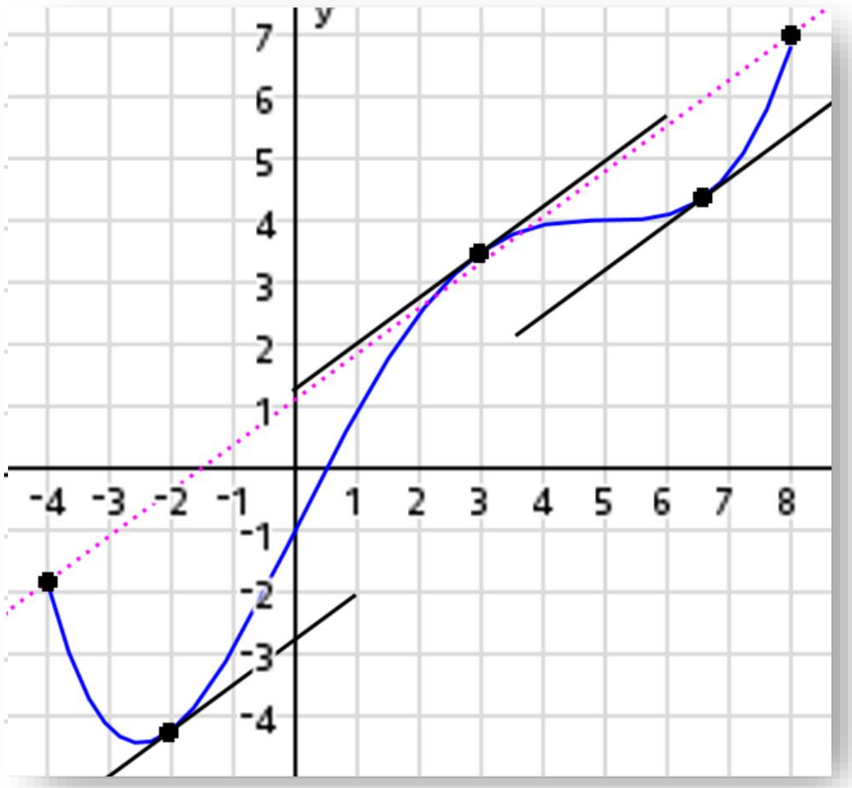
Justify your answer…

Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied.

– Stephen Davis

Multiple Choice

The graph of a differentiable function *f* is shown on the closed interval [-4,8]. How many values of x in the open interval (-4,8) satisfy the conclusion of the Mean Value Theorem for *f* on [-4,8] ?



(A) None

(B) One

(C) Two

(D) Three

Multiple Choice

x	2	5	6	10	15
$f(x)$	0	1	-2	10	13

The table above gives selected values for the differentiable function f. In which of the following intervals, must there be a number c such that $f'(c) = 3$?

(A) $(2, 5)$ (B) $(5, 6)$ (C) $(6, 10)$ (D) $(10, 15)$

Multiple Choice

Let f be the function given by $f(x) = 2x + 4\cos(x)$. For what value of x in the closed interval $\left[0, \frac{2\pi}{3}\right]$ does the instantaneous rate of change of f equal the average rate of change of f?

(A) 0.798
(B) 0.865
(C) 1.185
(D) 1.353

Mean Value Theorem

If g is a continuous function on the closed interval $[a, b]$ and $f(x) = \int_{\xi}^{x} g(t)\,dt$, then by the Fundamental Theorem of Calculus, we have…

1) $f'(x) = g(x)$
2) $g = f'$ is cont $\rightarrow f$ is diff $\rightarrow f$ is cont
3) $\int_{a}^{b} g(x)\,dx = \int_{a}^{b} f'(t)\,dt = f(b) - f(a)$

And so…

f is continuous,
f is differentiable

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

g is continuous

$$g(c) = \frac{\int_a^b g(x)dx}{b-a}$$

Mean Value Theorem for Integral

If a function g is continuous over the closed interval $[a, b]$, then there exists some c, $a < c < b$, such that

$$g(c) = \frac{\int_a^b g(x)\,dx}{b - a}.$$

MVT Integral MCQ

The rate at which water leaks from a leaky cauldron can be modeled by the function $R(t) = \sqrt[3]{36 - t^2}$ where t is measured in hours and $R(t)$ is measured in liters per hour. What is the average rate at which water leaks from the cauldron between $t = 0$ and $t = 4$ hours in liters per hour?

(A) -0.147
(B) 2.714
(C) 3.008
(D) 3.121

FRQ

A swimming pool is being emptied. The rate at which water is being pumped out of the pool is modeled by a differentiable function p, where $p(t)$ is measured in gallons per minute and t is measured in minutes since pumping began. Selected values of $p(t)$ are in the table.

t minutes	0	60	180	240	360	480
p(t) Gallons per minute	0	6	14	20	25	32

Use a right Reimann sum to approximate $\frac{1}{480}\int_0^{480} p(t)\,dt$. Interpret the meaning of this integral in context of this problem.

$$\frac{1}{480}\left(p(60)\cdot 60 + p(180)\cdot 120 + p(240)\cdot 60 + p(360)\cdot 120 + p(480)\cdot 480\right)$$

$$= \frac{1}{480}\left(6\cdot 60 + 14\cdot 120 + 20\cdot 60 + 25\cdot 120 + 32\cdot 480\right)$$