Math Objectives

- Students will recognize that a system of two equations in two variables can have no solution, one or more solutions, or infinitely many solutions.
- Students will determine whether a graph is a function by using the vertical line test.
- Students will determine the number of possible real roots by using a moveable horizontal line.
- Students will attend to precision (CCSS Mathematical Practice).

Vocabulary

- system of equations
- root or zero of a function
- intersection
- vertical line test
- infinitely many solutions
- coinciding figures

About the Lesson

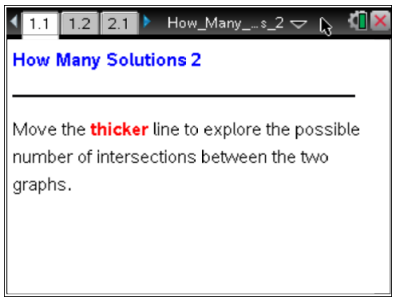
- Students will begin the lesson by manipulating a moveable line in the coordinate plane in relation to a fixed line. Students will discover what must be true for a system of linear equations to have one, infinitely many, or no solutions.
- Students will continue the lesson by manipulating a moveable line in the coordinate plane in relation to graphs of various fixed relations.
- They will rotate the moveable line until it appears to be vertical on each page. Students will translate the line across the screen to determine if the graph is a function.
- Students will rotate the moveable line until it appears to be horizontal on each page. Students will translate the line up and down the screen to determine the possible number(s) of real roots of the graph.
- Given different systems, students will illustrate the possible number(s) of solutions.

TI-Nspire™ Navigator™

- Use Quick Poll to assess students' understanding.
- Use Class Capture or Live Presenter to monitor student progress.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- How_Many_Solutions_2_Student.pdf
- How_Many_Solutions_2_Student.doc

TI-Nspire document

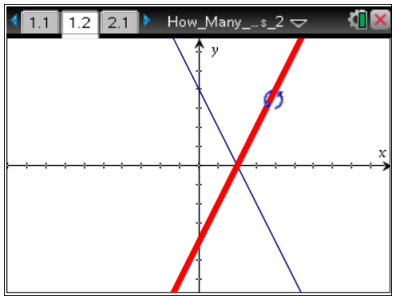
- How_Many_Solutions_2.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty rotating the line (↻) or translating the line (✣), check to make sure that they have moved the cursor (arrow) until it becomes ↻ or ✣. Then press ctrl to grab the line and close the hand. When finished moving the line, press esc to release the line. Note that only one line is moveable.

Tech Tip: To move a line graph, grab and hold either end of the line to rotate it. Grab and hold the middle of the line graph to move the entire line.

Move to page 1.2.



1. Move the thicker line to show all possible numbers of solutions to the system of equations. Sketch each possibility below.

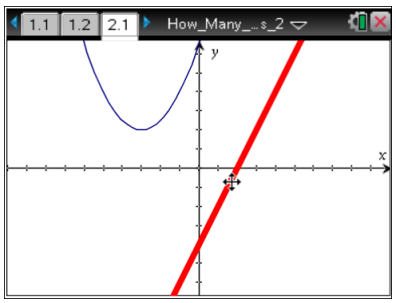
   Answer: Students should have examples that include the lines having exactly one solution (one point of intersection), no solution (lines are parallel), and infinitely many solutions (lines coincide).

2. Given two lines, in how many points can they intersect?

   Answer: one, none, or infinitely many

   Teacher Tip: You may want to discuss how the slopes of the lines indicate whether a system is independent, dependent, consistent, and/or inconsistent.

Move to page 2.1.



3. Move the line to show all possible numbers of solutions to the system of equations. Sketch each possibility below.

   Answer: Students should have sketches that include the line intersecting the parabola at two points, one point (the line is tangent to the parabola), and no points (the line never intersects the parabola).

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 1 at the end of this lesson.

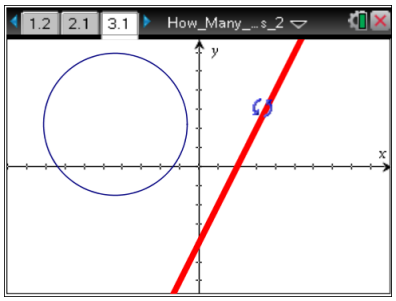
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

4. Given a parabola and a line, in how many points can they intersect?

   Answer: one point, two points, or no points

   Teacher Tip: Be sure the students take into account the graphs extending beyond the screen. Some students may think the graphs do not intersect at all because they appear to not intersect on the screen, when they may actually intersect at a point beyond the window.

Move to page 3.1.



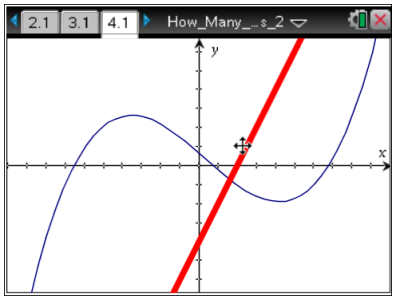
5. Move the line to show all possible numbers of solutions to the system of equations. Sketch each possibility below.

   Answer: Students should have sketches that include the line intersecting the circle at two points, one point (the line is tangent to the circle), and no points (the line never intersects the circle).

6. Given a circle and a line, in how many points can they intersect?

   Answer: one point, two points, or no points

Move to page 4.1.



7. Move the line to show all possible numbers of solutions to the system of equations. Sketch each possibility below.

   Answer: Students should have sketches that include the line intersecting the cubic at three points, two points, and one point.

Tech Tip: If students need to quickly change the window, help them by moving the cursor to white space and pressing ctrl to get the hand. Use the arrow keys to quickly change the window. Pressing ctrl Z will return the window back to the original size.

8. Given a cubic and a line, at how many points can they intersect?

   Answer: one point, two points, or three points

   Teacher Tip: Because this is an odd-degree polynomial function, there will always be at least one point of intersection.

9. A real root exists at the point where a graph intersects the *x*-axis.

   a. On page 1.2, rotate the thicker line until it appears to be horizontal. Consider this line to be a movable *x*-axis. Translate the line up and down to determine the possible numbers of real roots. How many possible real roots exist?

      Answer: Exactly 1 real root exists.

   b. Repeat this process for page 2.1. How many possible real roots exist?

      Answer: There are 2, 1, or 0 real roots.

   c. Repeat this process for page 3.1. How many possible real roots exist?

      Answer: There are 2, 1, or 0 real roots.

   d. Repeat this process for page 4.1. How many possible real roots exist?

      Answer: There are 3, 2, or 1 real roots.

10. How do the answers in question 9 relate to the answers in questions 1–8?

    Answer: The answers are the same, which means that the possible number of real roots of a relation is the same as solving for the possible numbers of solutions of the system of equations when one of the equations is $y = 0$.

11. The vertical line test is used to determine whether a relation is a function. If a vertical line passes through more than one point on the graph of a relation, the relation is not a function.

    a. On page 1.2, rotate the thicker line until it appears to be vertical, and then translate it from left to right across the screen. Is the graph a function? Explain.

    Answer: Yes. As the vertical line moves across the screen, it never intersects the other line at more than one point.

    b. Repeat this process for page 2.1. Is the graph a function? Explain.

    Answer: Yes. As the vertical line moves across the screen, it never intersects the parabola at more than one point.

    c. Repeat this process for page 3.1. Is the graph a function? Explain.

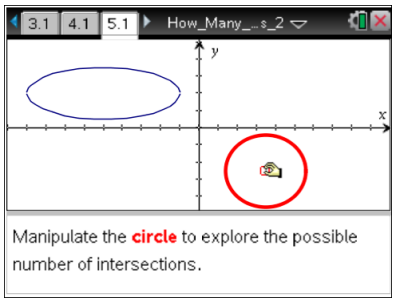
    Answer: No. There are several times when the vertical line intersects the circle at more than one point.

    d. Repeat this process for page 4.1. Is the graph a function? Explain.

    Answer: Yes. As the vertical line moves across the screen, it never intersects the cubic at more than one point.

> **Teacher Tip:** Caution students that, although the vertical line appears to touch the cubic in more than one spot toward what appears to be the ends of the cubic, it actually does not. The teacher may want to change the window to zoom out in order to show that the vertical line never intersects the function at more than one point.

Move to page 5.1.



12. Drag the point to move the circle to show all possible numbers of solutions to the system of equations. Sketch each possibility below. (It is possible to change the diameter of the circle.) Use one screen to show more than one answer if necessary.

    Answer: Students should have sketches that include the circle intersecting the ellipse at four points, three points (the circle is tangent to the ellipse at one point), two points, one point, and no points (the circle never intersects the ellipse).

Tech Tip: Students may have trouble moving the circle because it will change the radius of the circle if they try to grab the circle itself. To move the circle, they must grab the center by moving the arrow until it becomes a hand (✋) getting ready to grab the center. Also, be sure that the word *point* appears. Then press ctrl ✋ to grab the point and close the hand (✊).

13. Given the following systems, determine all possible numbers of solutions:

    a. the graphs of a circle and a parabola

    Answer: There are 4, 3, 2, 1, or no solutions.

    b. the graphs of two circles

    Answer: There are infinitely many solutions (the circles coincide), 4, 3, 2, 1, or no solutions.

    c. the graphs of two parabolas where one parabola is sideways and the other opens up

    Answer: There are 4, 3, 2, 1, or no solutions.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- That there are various numbers of solutions to systems of equations.
- That the vertical line test can be used to determine whether a graph is a function.
- That the solution to a system of equations where one of the equations is $y = 0$ can be called a root (zero) of the equation.
- How to use visualization to determine the possible number(s) of solutions to a system of equations.

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Note 1

Question 3, *Class Capture*: Use Class Capture for students to see all the different ways that the line will intersect the parabola. This will enable them to more readily see that it can intersect in one, two, or no places. Repeat the Class Capture for questions 5 and 7.

Note 2

Entire Document, *Quick Poll*: Use Quick Poll (Open Response) and ask students to respond to any of the questions.