Pythagoras – Episode 3

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Student Activity

7 8 **9** 10 11 12

TI-Nspire | Investigation | Student | 30 min

Introduction

The first two stages in this journey have probably raised as many questions as they have answered. Pythagoras's theorem has thrown us into a numerical world that did not previously exist; numbers that cannot be fully tamed or expressed have a strangle hold on our synapses.

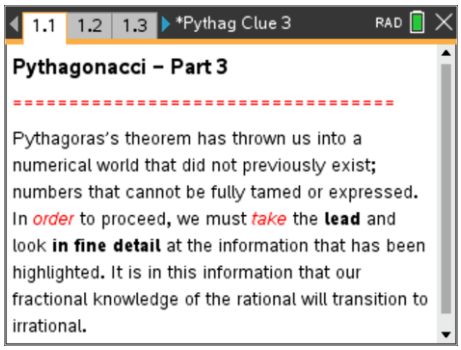
In *order* to proceed, we must *take* the **lead** and look **in fine detail** at the information that has been highlighted. It is in this clue that our fractional knowledge of the rational will transition to irrational. Once you have cracked the clue the *code will be revealed* and the next stage of your journey will become clearer.

Continued Fractions

Open the TI-Nspire file: **Pythag Clue 3**

The first page of the document contains some important text from above. Read it carefully! If you can figure out the answer, store it on page 1.2 where it currently says clue:= "empty".

If you cannot solve the riddle, you can still progress to page 1.3.



If you have an answer to the riddle, it must be placed in quotation marks as per the sample provided on page 1.2. The simple way to do this is to copy and paste the sample.

Page 1.3 contains a calculator application, ready for some calculations.

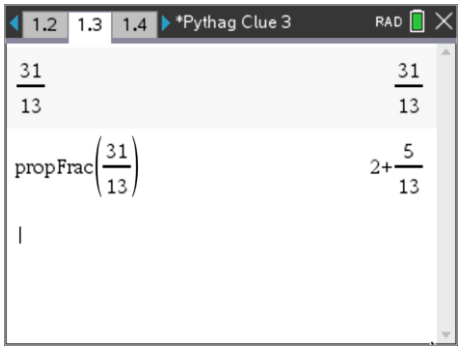
The fraction key on TI-Nspire can be accessed by pressing: ctrl + ÷

By default, a calculation such as 31 ÷ 13 will be expressed as an improper fraction. [See opposite]

This can be changed to a mixed fraction by pressing:

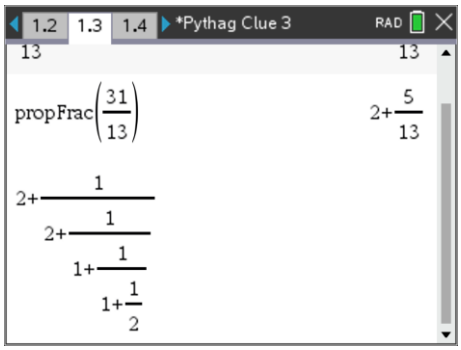
menu > **Number** > **Fraction Tools** > **Proper Fraction**

The previous calculation can be copied and pasted into the propFrac command or simply re-typed.



This is not the only way to represent a fraction in mathematics.
A continued fraction provides a different way of looking at the 'remainder' portion: 5 ÷ 13.

Use your calculator to write the fraction shown below and opposite).
You will need to use the fraction template: ctrl + ÷ and the tab or arrow keys to navigate.



$$2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}$$

Author: P. Fox

Question: 1.

In reference to the continued fraction shown above:

a) What answer does your calculator produce?

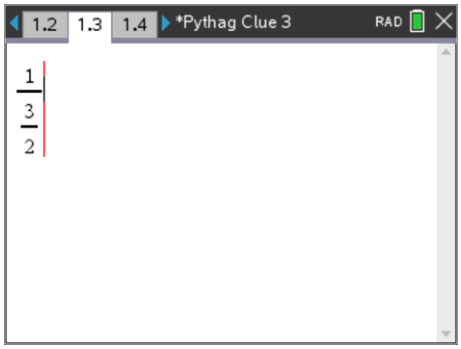
b) If you had to do this question by hand (no calculator), where would the first calculation be?

c) If you had to do this question by hand (no calculator), without actually doing the calculation, where would the second calculation occur?

To help understand how these continued fractions are created, we first need to understand why the sequence of '1's appears in the numerator of each 'sub fraction'.

Express the fraction shown opposite on your calculator, notice that the vinculum (line separating the numerator and denominator) is larger at the top. This means the expression is interpreted as: $1 \div (3 \div 2)$.

Try some examples of your own, each must be of the form: $1 \div (a \div b)$.



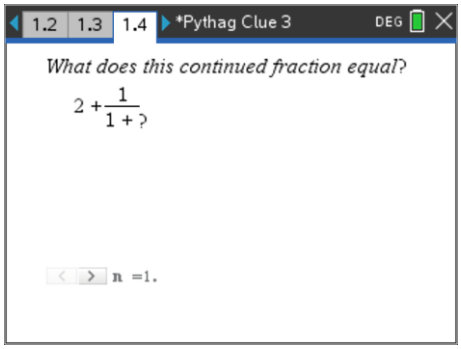
Question: 2.

Based on your answers to the previous calculator task, express the following as simple fractions:

a) $\dfrac{\frac{1}{5}}{6}$ b) $\dfrac{\frac{1}{3}}{7}$ c) $\dfrac{1}{\frac{13}{121}}$

Navigate to page 1.4 and use the slider (spinner) to progressively reveal or hide sections of the fraction. The ability to hide will be useful when performing the calculations by hand.

$$2+\cfrac{1}{1+\cfrac{1}{5+\frac{1}{2}}}$$



Question: 3.

Use spinner on page 1.4 to reveal the entire continued fraction, then calculate each portion, one at a time.

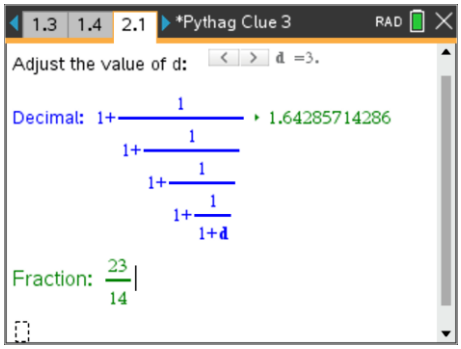
a) What calculation needs to be performed first? Express your calculation as an improper fraction.

b) What calculation needs to be performed second? Express your calculation as an improper fraction.
Note: You will need your answer from part (a) first. Consider adjusting the slider accordingly.

c) What calculation needs to be performed last? Express your calculation as an improper fraction.
Note: You will need your answer from parts (a & b) first. Consider adjusting the slider accordingly.

Author: P. Fox

Navigate to page 2.1, this is a Notes application. A continued fraction is displayed. The value of (d) is displayed at the top of the screen. Changing this value will automatically generate both the decimal and fraction answers for the continued fraction displayed.



If the values for the calculations are not displayed in the Notes application, press menu > **Actions** > **Activate All**

Question: 4.

Use the slider to change the value of (d) and record the results for each value, decimal and fraction.

d	0	1	2	3	4	5	6	7	8	9
Decimal:										
Fraction:										

What do you notice about the decimal values?

Question: 5.

Navigate to page 3.1, another version of this continued fraction is included, this time without the fractional result and more sub-fractions. Use the slider to change the value of (d) and record the results for each value.

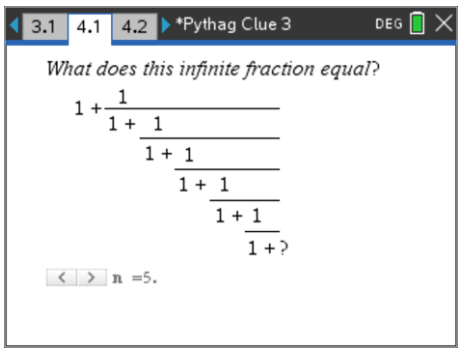
d	0	1	2	3	4	5	6	7	8	9
Decimal:										

What do you notice about the decimal values now that there are more 'sub-fractions'?

Infinite Continued Fractions

Navigate to page 4.1. Use the slider to progressively reveal the fraction.

How can we calculate this fraction if it continues … for ever?



Author: P. Fox

There is another way to approach this problem. Notice that for each section of the continued fraction, the (?) is replaced with the expression:

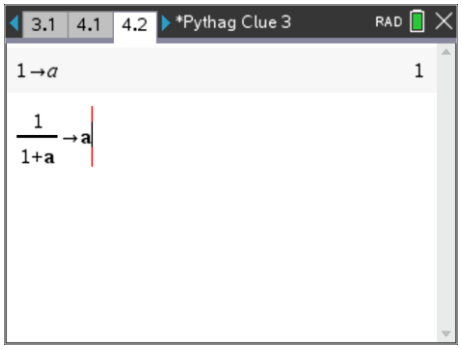
$$\frac{1}{1+?}$$

In mathematics we can use a variable to represent an unknown.

Navigate to page 4.2 (Calculator application). We need to seed '*a*' with a value. To store *a* as 1, press:

1 ctrl + var A enter

Now store the expression into *a*. [As shown opposite]



Question: 6.

Once the expression has been stored, keep pressing enter to see what happens. To calculate the approximate (≈) result press: ctrl + enter.

What is the approximate value for the infinite fraction as it continues … on and on? [Fraction on page 4.1]
Note: You will need to add the original '1' from the very first line.

Question: 7.

Navigate to Page 5.1, it contains a different 'infinite' fraction. Explore the approximate value for this infinite fraction.

Question: 8.

With the assistance of the following, determine a simple fraction that provides an excellent estimation for this infinite fraction.

Use the slider to progressively remove each sub-fraction, notice that a value has been inserted to replace the (?) used by the infinite fractions.

Step 5: $1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{3}}}}}}$ $\therefore ? = \frac{1}{3}$

Step 4: ? = Step 3: ? = Step 2: ? = Step 1: ? =

Express the final result as an improper fraction and a decimal.

Question: 9.

Once you have identified the irrational number represented by the previous infinite fraction. Try and determine an infinite fraction for a different square-root value.

Explore

An interesting infinite fraction is of the form:

$$\cfrac{4}{1+\cfrac{1^2}{2+\cfrac{3^2}{2+\cfrac{5^2}{2+\cfrac{7^2}{2+\cfrac{9^2}{2+\ldots}}}}}}$$

This infinite fraction has been set up in a program. Navigate to page 6.1 to see this fraction, then to page 6.2.

A program has been set up to start at the bottom of this fraction and work up … you can observe each row as it is computed. To run the program type: secrete() then press enter .
You will be prompted for the number of iterations (sub-fractions) you want to execute, up to 100. What number is being generated by this infinite fraction?

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