Building Concepts: What is an Equation?

Lesson Overview

Algebraic Focus: How do equations differ from expressions? What does it mean to have a solution to an equation?

In this lesson students begin by establishing the difference between an expression and an equation. Students then investigate expressions and equations to develop what it means to have a solution to an equation.

In an equation, a constraint is imposed such that the values of the variables involved in the expressions on each side of the equation must make the equation a true statement.

Learning Goals

1. Distinguish between an expression and an equation;
2. recognize that solutions to an equation are the replacement values for the variable that make the equation true;
3. identify equations that have no solutions, a finite number of solutions or an infinite number of solutions;
4. reason about the nature of the solutions to an equation.

Prerequisite Knowledge

What is an Equation? is the fourth lesson in a series of lessons that explore the concept of expressions. This lesson builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *What is an Exponent? What is a Variable?* and *Building Expressions*. Students should understand:

- the order or operations;
- the associative and commutative properties of addition and multiplication;
- how to verify that expressions are equivalent.

Vocabulary

- **expression:** a phrase that represents a mathematical or real-world situation
- **equation:** a statement in which two expressions are equal
- **variable:** a letter that represents a number in an expression
- **solution:** a number that makes the equation true when substituted for the variable

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- What is an Equation_Student.pdf
- What is an Equation_Student.doc
- What is an Equation.tns
- What is an Equation_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice, and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

As described in Activity 3, *Building Expressions*, an expression is a phrase about a mathematical or real-world situation. An *equation* is a statement that two expressions are equal, such as $10+5n=20$, or $5+x=4+x$. Note that the two expressions on either side of the equal sign might not actually always be equal; that is, the equation might be a true statement for some values of the variable(s) and a false statement for others. For example, $10+5n=20$ is true only if $n=2$, and $5+x=4+x$ is not true for any number *x*, while $2x+6=2(x+3)$ is true for all possible values of *x*. A *solution* to an equation is a number that makes the equation true when substituted for the variable (if there is more than one variable, a number for each variable will make the equation true). An equation may have no solutions. For example, $5+x=4+x$ has no solution because, no matter what number *x* is, it is never true that adding 5 to *x* can give the same answer as adding 4 to *x*. An equation that is true no matter what number the variable represents is called an *identity* (e.g., $2x+6=2(x+3)$); however identity is not used as a term in this lesson.

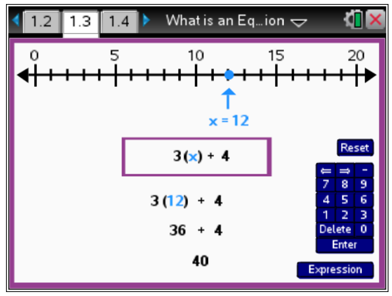
Students often confuse expressions and equations and “finish” expressions by setting them equal to 0 and solving. A major focus of this activity is to help students make the distinction between the two concepts. Expressions represent a description of mathematical calculations involving numbers and/or variables. Equations represent a relationship between these descriptions. In an equation, a constraint is imposed such that the values of the variables involved in the expressions on each side of the equation must make the equation a true statement.

The focus of this lesson is not on learning how to solve an equation but on what an equation is and what it means to have a solution to an equation. As students begin their work with algebraic equations, they should learn to think and reason about the nature of the solutions: what values would make sense both numerically and in a given context (e.g, would the solution have to be an even number? can a person grow negative 4 inches per year?), and how different equations representing the same situation can be related to each other (e.g., how is $x+2=8$ different from and the same as $x+4=10$?).

Note: the questions for pages 1.3 and 1.5 are designed for students without experience with negative integers. Operations with signed numbers are in Grade 7, so in this lesson the main focus is on expressions and equations that do not involve negative rational numbers. If students have had experience with negative integers, the questions can be adapted for use with pages 2.2 and 2.3, where the number line goes from -10 to 10.

Part 1, Page 1.3

Focus: Students explore what is meant by an equation and the solution to an equation. Students also distinguish between an expression and an equation.



On page 1.3, students can drag the dot or use the arrows to change the value of *x* to a whole number between 0 and 20.

Numbers allows the Multiplier/Constant to be edited.

Operation allows the operation to toggle between addition and subtraction.

Expression/Equation toggles between an expression and equation.

Use the left and right arrows to change the value of the variable.

TI-Nspire Technology Tips

[menu] accesses page options.

[tab] toggles through the multiplier, constant, and total.

[+], [−] changes the operator.

[enter] toggles the expression/ equation button.

[ctrl] [del] resets the page.

Class Discussion

The following questions explore what is meant by an equation and the solution to an equation while ensuring that students can distinguish between an expression and an equation

Have students…	Look for/Listen for…
Write down everything you know about variables and expressions. Use examples from page 1.3 to illustrate your thinking.	Answers will vary. Students should be able to note that a variable is typically represented by a letter and takes on multiple values as replacements. An expression is a mathematical phrase involving numbers, variables, mathematical operations, and mathematical symbols such as parentheses.
Use the file to help answer each.	
• ***When you use the file, what are the possible values for the variable?***	Answer: Whole numbers from 0 to 20 or *x* is a whole number such that $0 \le x \le 20$. Students should be encouraged to use symbols and words to describe this set.
• ***Is there any value for the variable for which the expression will equal 31?***	Answer: When $x = 9$, the expression will equal 31.

Class Discussion (continued)

- ***What is the maximum value for the expression you can find using whole numbers between 0 and 20 as replacements for the variable?***

  Answer: 64

Change the numbers to create an expression that satisfies each of the following. Be ready to share your thinking with the class.

- ***65 when x is 7***

  Answers will vary. One possible response is $3x+44$ because when I move x to 7 for $3x+4$ the value of the expression was 25. This was 40 short of 65, so I changed the constant to 44. Another response might be $5x+30$ because I know $5(7)=35$, so I need to add 30 to get 65.

- ***250 when x is 9***

  Answers will vary. Examples: $20x+70$ because $9(20)=180$ and I need 70 more to get to 250; $25x+25$ because $25(10)=250$ and $25(9)$ would be 25 short so you would add 25 to $25(9)$

- ***0 when $x=17$***

  Answers will vary. Examples: $1x-17$ because $17-17=0$; $2x-34$ because when you subtract the same number as the product of $2(17)$, which is 34, from $2(17)$ you will get 0.

Reset page 1.3 and select Equation.

- ***Describe the difference between the new screen and the previous one.***

  Answer: This shows an expression with an equal sign and a number to the right of the equal sign. The page before only had the expression on the left side of the equals sign.

- ***Drag the dot until the left side of the equation matches the right side. Explain what this means.***

  Answer: When $x=8$, the value of the expression on the left side of the equals is 28, the same as the number on the right side.

- ***How many values of x make the statement true. Justify your answer.***

  Answer: Only one value for x because for $x>8$, the values of the expression get larger, increasing 3 at a time away from 28, and for $x<8$, the values of the expression get smaller decreasing 3 at a time. So the only value for which the expression is equal to 28 is when $x=8$.

Class Discussion (continued)

Select Expression. Create the expression $15x - 30$. Which of the following statements make sense for this expression? Explain your reasoning in each case.

• ***$x = 2$ will make the expression true.***	Answer: This does not make sense because *x* can be any number in an expression, not just 2.
• ***When $x = 3$, the value of the expression will be 15.***	Answer: This makes sense because for $x = 3$, $15(3) - 30 = 45 - 30 = 15$.
• ***The only value for the expression $15x - 30$ is 0.***	Answer: This does not make sense because the expression does not have just one value. One way to say this is to say an expression is not "constrained" to have values for the variable that make the expression true as is the case for an equation.
• ***As the value of x decreases, the value of $15x - 30$ decreases.***	Answer: This makes sense because as *x* decreases by 1, the value of the expression decreases by 15.

Select Equation. Using the equation $15x - 30 = 0$ instead of the expression $15x - 30$, how would your answers to the set of questions above change?

• ***$x = 2$ will make the equation true.***	Answer: Now this makes sense because $x = 2$ is the solution and makes a true statement: $15(2) - 30 = 0$.
• ***When $x = 3$, the equation equals 15.***	Answer: $x = 3$ makes the value of the left side of the equation 15, $15(3) - 30 = 45 - 30 = 15$, but the equation is given as the statement that $15x - 30 = 0$ and $15 \neq 0$.
• ***As the value of x decreases, the value of $15x - 30 = 0$ decreases.***	Answer: This does not make sense because an equation does not have a numerical "value" and cannot increase and decrease; it is a statement of equality.

Student Activity Questions—Activity 1

1. **Reset page 1.3. Create each equation, then identify the values of *x* that make the equation true. Explain how you know you have all values that could be a solution.**

   a. **$12-18=54$**

   Answer: $x=6$ because $12(6)-18=54$ is true. There are no other solutions because for every value larger than 6, the value of the expression on the left is larger than 54 and for every value less than 6, the value of the expression on the left is less than 54.

   b. **$0x-2=5$**

   Answer: No values of *x* make the equation true. Any number times 0 is 0 and $0-2\neq 5$.

   c. **$0x+4=4$**

   Answer: All values of *x* make the equation true because the product of any number and 0 is 0; $0+4=4$ so any value of *x* will make a true statement.

2. **A *solution* to an equation is the number for the variable that makes the equation a true statement.**

   a. **What are the possible numbers for the variable x for any equation you create on page 1.3?**

   Answer: Whole numbers from 0 to 20 are the possible values for the variable x.

   b. **Is there a solution for $8x+16=24$ in the numbers you gave for 2a? Explain why you know you have the solution.**

   Answer: The solution is $x=1$ because 1 makes the equation a true statement.

   c. **Is there a solution for the equation $8x+2=6$ in the set of values you gave for 2a?**

   Answer: There are no whole number solutions for this equation. When $x=0$, the expression $8x+2=2$ and when $x=1$, the expression $8x+2=10$. So, the solution is not a whole number. ($x=\frac{1}{2}$ would be a solution if $\frac{1}{2}$ was possible to use for *x*.)

3. **Identify the solution for each equation below. Explain how you know there are no other solutions.**

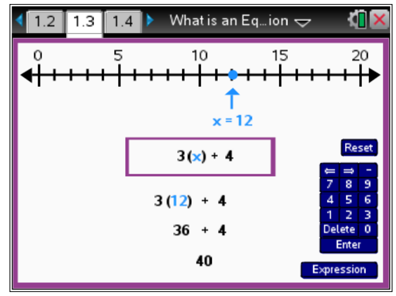
   a. $19x+9=218$ b. $12x+12=12$ c. $24x=24$

   Answers: a. $x=11$, b. $x=0$, c. $x=1$

   Reasons may vary. One response might be: You have all of the solutions because in each case as the numbers for x become larger than the solution, the value of the expression increases, and when the numbers for *x* become smaller than the solution, the value of the expression decreases.

Part 2, Page 1.3

Focus: Students reason about the possible number of solutions for certain equations and what conditions might lead to a finite number of unique solutions, no solutions, or all possible values of the variable as solutions. They also develop equations in contextual situations and decide whether the equations they create lead to reasonable solutions.



Class Discussion

Have students…

For each of the following pairs determine whether the same value of x is the solution to both. Explain your reasoning, then use the file to check.

- ***$10x - 20 = 30$ and $x - 2 = 3$***

  Answer: $x = 5$ makes both statements true. This makes sense because everything in the first equation is 10 times the terms in the second equation.

- ***$7x - 19 = 44$ and $7x = 63$***

  Answer: $x = 9$ for both. This makes sense because 19 added to both sides of the first equation creates the second equation.

- ***$4x + 31 = 75$ and $4x + 25 = 69$***

  Answer: $x = 11$ for both. This makes sense because taking six away from both sides of the first equation creates the second equation.

- ***$3x - 6 = 42$ and $x - 6 = 14$***

  Answer: The solution for the first equation is 16, and for the second equation it is 20.

Look for/Listen for…

Student Activity Questions—Activity 2

1. Angelo's pet rabbit weighs twice the weight of Carmen's pet rabbit.

a. Does Carmen's or Angelo's rabbit weigh the most?

Answer: Angelo's rabbit

b. Angelo's pet rabbit weighs 12 pounds. Write an equation that could be used to find the weight, *w*, of Carmen's rabbit. Use the equation to find the weight of Carmen's rabbit. Does your answer seem reasonable? Why or why not?

Answer: $2w = 12$; $w = 6$ pounds so Carmen's rabbit weighs 6 pounds, and that seems like a reasonable weight.

Student Activity Questions—Activity 2 (continued)

c. **Angelo's pet rabbit weighs 2 pounds less than twice the weight, *x*, of Simon's pet rabbit. Which rabbit weighs the most, Carmen's or Simon's? Explain your thinking.**

Answer: Simon's rabbit weighs more than Carmen's because Angelo's rabbit is less than twice the weight of Simon's rabbit and is twice the weight of Carmen's rabbit.

d. **Create each of the following equations using the file and find the solutions. Which equation seems reasonable to find the weight of Simon's rabbit and gives a reasonable solution?**

i. $12 = 2x + 2$ **ii.** $12 = 2x - 2$ **iii.** $x - 12 = 2$ **iv.** $w - 2 = 12$

Answer: Solutions: i) 5 lbs; ii) 7 lbs; iii) 14 lbs; and iv) 14 lbs. Angelo's rabbit has to weigh more so iii and iv are not reasonable equations; Equation i is not correct because it added the 2 pounds to twice the weight rather than subtracting and the solution gives a number less than the weight of Carmen's rabbit. Equation ii is correct because it accounts for the 2 pounds less.

2. **Decide whether the following statements are true or false. Give an example to support your thinking. (You may want to use the file to find an example.)**

a. **Some equations have more than one solution.**

Answer: True, for example, $0x + 10 = 10$. Examples may vary.

b. **Some equations do not have any solutions.**

Answer: True, for example, $0x + 10 = 10$. Examples may vary.

c. **Some expressions have an infinite number of solutions.**

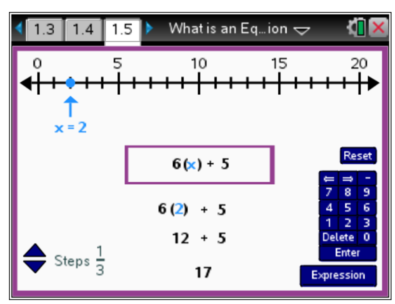
Answer: False because an expression is not an equation and so does not have a solution. $2x + 4$ is not supposed to equal any given outcome, so *x* can have any value. Examples may vary.

d. **The values of some expressions will always be even numbers.**

Answer: True, for example $2x + 10$ will be even for any value of *x* because the product of 2 and any whole number is even and the sum of two even numbers will be even. Examples may vary.

Part 3, Page 1.5

Focus: Students reason about equations that have positive rational solutions. They need to recall operations with fractions and mixed numbers and are asked to apply their understanding in this new space.



Page 1.5 has the same operations as page 1.3, except the steps can be changed to fractional increments.

Steps allows the variable to change in fractional increments by using the up and down arrows.

Class Discussion

Have students…	Look for/Listen for…
Drag the dot.	
• ***What are possible values for the variable x on page 1.5?***	Answer: Any fraction of the form $\frac{a}{3}$ where *a* is a whole number.
• ***Click to Expressions. Find the solution for the equation.***	Answer: $x = \frac{2}{3}$
• ***Change the fraction to $\frac{1}{6}$. Can you find a solution for the equation in this set? Explain why or why not.***	Answer: Yes, the fraction equivalent to $\frac{2}{3}$, $\frac{4}{6}$,will be a solution.
On page 1.5, create an equation that has the given solution. Be ready to explain your thinking to the class.	
• ***The solution is $x = 7\frac{2}{3}$ and the value on the right side of the equation is 37.***	Answers will vary. One possible response is $6x - 9 = 37$. The multiplier should be a multiple of 3 because the rest of the numbers are whole numbers; if you use 6 as the multiplier, then $6\left(7\frac{2}{3}\right) = 46$ and you need to subtract 9 to get to 37.
• ***The solution is $6\frac{1}{5}$, and the value on the right side of the equation is 37.***	Answers will vary. One possible response is $5x + 6 = 37$ because you need a multiplier of 5 to cancel out the fraction in the solution and $5\left(6\frac{1}{5}\right) = 31$, so you need 6 more to get to 37.
• ***The solution is $12\frac{3}{8}$ and the constant term on the left side of the equation is 99.***	Answers will vary. One possible response is $16x - 99 = 99$ because the product of 16 and $12\frac{3}{8}$ a whole number, 198. And you have to subtract 99 from that so the right side of the equation is also 99.

Student Activity Questions—Activity 3

1. Write an expression that represents each situation. Let *x* be the age of Sol's brother

a. Sol is one year away from being twice as old as his brother.

Answer: Both $2x - 1$ and $2x + 1$satisfy the constraint.

Student Activity Questions—Activity 3 (continued)

b. **Summer is three years older than Sol's brother.**

Answer: $x+3$ represents Summer's age.

c. **Sol's dad is five times as old as Sol's brother.**

Answer: 5*x* represents the age of Sol's dad.

d. **Sol's mom is 5 years younger than Sol's dad.**

Answer: $5x-5$ represents the age of Sol's mom.

2. **If Sol is 17 years old, use your work in the question above and write equations to help you find the ages of his mom, dad, brother and sister. You may use the file to solve the equations.**

a. **Find Sol's brother's age.**

Answer: $2x-1=17$ or $2x+1=17$, so Sol's brother would be either 8 or 9 years old.

b. **Find the ages of Summer, Sol's dad, and Sol's mom.**

Answer: Summer is $x+3$, where $x=8$ or 9 so she would be either 11 or 12 years old. His dad is 5*x* years old, so his dad is either 40 or 45 years old, and his mom is $5x-5$ so she would be 35 or 40.

3. **A ski resort had a 27-inch snow base at the beginning of the week and six days later it had 42 inches of snow. Let *x* represents the average number of inches of snow per day over a six-day period.**

a. **What does 2*x* represent? 6*x*?**

Answer: 2*x* represents the amount of snow in two days. 6*x* represents the amount of snow that fell in six days.

b. **Which of the equations $6x=42$, $6x+27=42$ and $6x-27=42$ seem reasonable to represent the total amount of snow at the resort at the end of the sixth day? You might want to use the file to help you figure out your answer. Explain your reasoning.**

Answers. For $6x=42$, $x=7$ inches, which is reasonable but ignores the 27-inch base. For $6x-27=42$, $x=\frac{23}{2}$ and you would have over 66 inches of new snow but this was on top of the old snow, so you would have to add the 27, not subtract it to get to 42. The right equation would be $6x+27=42$ because you would have the new snow plus the base to equal the total amount of snow.

Student Activity Questions—Activity 3 (continued)

c. **The six-day report for four other ski resorts is in the chart. Which resort do you think had the greatest average amount of snow over the six days? The least amount? Explain your reasoning.**

Resort	Snow base (inches)	Average snowfall per day (inches)	Depth at end of six days (inches)
Ski Run	32		54
DownHill	25		40
Creekside	34		48
Mountain Run	24		32

Answer: The biggest difference between the base and the end amount of snow was for Ski Run with a 22-inch gain, so it has the highest average amount of snow per day, while the smallest difference was 8 inches for Mountain Run, so it would have the least average amount of snow per day.

d. **Use the file to help find the solution for the average amount of snow per day at each of the resorts. Explain how you would have to adjust your equations in each case.**

Answer: you would change the constant term on the left expression to the new snow base and the right side of the expression to the new depth.

Resort	Snow base (inches)	Average snowfall per day (inches)	Depth at end of six days (inches)
Ski Run	32	$3\frac{2}{3}$ inches	54
DownHill	25	$2\frac{1}{2}$ inches	40
Creekside	34	$2\frac{1}{3}$ inches	48
Mountain Run	24	$1\frac{1}{3}$ inches	32

Deeper Dive — Page 1.3

Reset page 1.3. Without changing the file, do you think the solution for each new equation will be larger or smaller than the solution, $x = 8$, for $3x + 4 = 28$? Explain your reasoning.

- ***Change the 4 to 12.***

  Answer: The solution will be smaller because you already have 8 more towards making the left side equal 28.

- ***Change the 3 to 2.***

  Answer: The solution will be larger because you are only doubling the value not multiplying by 3 to get to 28.

- ***Change the plus to a minus.***

  Answer: The value will be larger because you are taking away to get to 28 instead of adding on.

- ***Use the file to check your answers to the questions above.***

Sondra had to read 15 books in three months for class. Each month she wants to read one more book than she read the month before. She considers the following equations as possible to help her figure out how many books she has to read in each month. Let x = the number in the first month.

i. $3x = 15$ ***ii.*** $3x + 3 = 15$

iii. $x + 3 = 15$ ***iv.*** $15x = 3$

- ***Can you tell by looking at the equations if any of them are not likely to have a solution that is reasonable? Explain your reasoning.***

  Answers may vary. iii is not reasonable because you are adding the number of months, 3, to the number of books to get a number of books. iv is not likely to work because *x* has to be a whole number.

- ***Find the solutions to the equations and decide which of them satisfies Sondra's conditions. Be sure your solution makes sense.***

  Answer: In i, $x = 5$, but if she read 5 the first month and then 6 the next, and then 7 the third month, she would have read 18 books, not 15.

  ii is the right equation: $x = 4$. If she read 4 the first month, 5 the second, and 6 the third, she would have read 15 books. iii gives $x = 12$ but you cannot add 12 books and 3 months and get 15 books; iv would have a fraction, $\frac{1}{5}$, as the solution, which does not make sense for the context.

Deeper Dive —Page 1.3 (continued)

Which of the following equations can be solved using the file? Explain your thinking.

- $9 = 6x + 5$

  Answer: Yes, $9 = 6x + 5$ is only stating the equation $6x + 5 = 9$ in the opposite direction like $2 + 4 = 6$ is the same as saying $6 = 2 + 4$.

- $5 + 6x = 9$

  Answer: Yes, $5 + 6x$ is the same as $6x + 5$ because a change in the order of adding (commutative) does not change the result, so you can use $6x + 5 = 9$ to solve $5 + 6x = 9$.

- $3x + 7x + 3 = 48$

  Answers may vary. Some might see that this has $3x$ and a $7x$, while the file only has one multiplier with the x. Others may suggest that $3x + 7x = 10x$, so rewriting the equation as $10x + 3 = 48$ would give the same solution.

- $3(x + 2) = 77$

  Answers may vary. Some might indicate this is not in the right form so the file cannot be used. Others might suggest rewriting the expression on the left using the distributive property, and result will be an equation in the right form for the file.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Select all equations that have x=3 as a solution.

   a. $x+7=10$ b. $3+x=3$ c. $3x=1$ d. $4x=12$

   SBA Practice test, 2014

   Answer: a) $x+7=10$ and d) $4x=12$

2. In the morning Emily studied 20 minutes for a math exam. Later that evening Emily studied for x more minutes. Write an equation for the total number of minutes, y, that Emily studied for the math exam.

   SBA Practice test, 2014

   Answer: $y=20+x$

3. Robert has x books. Marie has twice as many books as Robert has. Together they have 18 books. Which of the following equations can be used to find the number of books that Robert has?

   a. $x+2=18$ b. $x+x+2=18$ c. $x+2x=18$ d. $2x=18$ e. $2x+2x=18$

   NAEP grade 8 2011

   Answer: c) $x+2x=18$

Use the following choices to answer questions 4, 5, and 6:

Adapted from NAEP, Grade 8 1992

a. None

b. One

c. Six

d. Seven

e. Infinitely many

4. If k can be replaced by any number, how many solutions can the expression $k+6$ have?

   Answer: a) None

5. If k can be replaced by any number, how many different solutions can the equation $k+6=0$ have?

   Answer: b) One

6. If k can be replaced by any number, how many different solutions can the equation $k-k=0$ have?

   Answer: e) Infinitely many

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Number Sold, *n*	4	0	5	2	3	6
Profit, *p*	$2.00	$0.00	$2.50	$1.00	$1.50	$3.00

7. Angela makes and sells special-occasion greeting cards. The table above shows the relationship between the number of cards sold and her profit. Based on the data in the table, which of the following equations shows how the number of cards sold and profit (in dollars) are related?

   a. $p = 2n$

   b. $p = 0.5n$

   c. $p = n - 2$

   d. $p = 6 - n$

   e. $p = n + 1$

NAEP, Grade 8 2007

Answer: b) $p = 0.5n$

Student Activity Solutions

In these activities you will identify solutions to equations. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Reset page 1.3. Create each equation, then identify the values of *x* that make the equation true. Explain how you know you have all values that could be a solution.

   a. $12-18=54$

   Answer: $x=6$ *because* $12(6)-18=54$ *is true. There are no other solutions because for every value larger than 6, the value of the expression on the left is larger than 54 and for every value less than 6, the value of the expression on the left is less than 54.*

   b. $0x-2=5$

   Answer: No values of x make the equation true. Any number times 0 is 0 and $0-2\neq 5$ *.*

   c. $0x+4=4$

   Answer: All values of x make the equation true because the product of any number and 0 is 0; $0+4=4$ *so any value of x will make a true statement.*

2. A *solution* to an equation is the number for the variable that makes the equation a true statement.

   a. What are the possible numbers for the variable x for any equation you create on page 1.3?

   Answer: Whole numbers from 0 to 20 are the possible values for the variable x.

   b. Is there a solution for $8x+16=24$ in the numbers you gave for 2a? Explain why you know you have the solution.

   Answer: The solution is $x=1$ *because 1 makes the equation a true statement*.

   c. Is there a solution for the equation $8x+2=6$ in the set of values you gave for 2a?

   Answer: There are no whole number solutions for this equation. When $x=0$ *, the expression* $8x+2=2$ *and when* $x=1$ *, the expression* $8x+2=10$ *. So, the solution is not a whole number (* $x=\frac{1}{2}$ *would be a solution if* $\frac{1}{2}$ *was possible to use for x.)*

3. Identify the solution for each equation below. Explain how you know there are no other solutions.

   a. $19x+9=218$ b. $12x+12=12$ c. $24x=24$

   Answers: a. $x=11$*, b.* $x=0$ *, c.* $x=1$

   Reasons may vary. One response might be: You have all of the solutions because in each case as the numbers for x become larger than the solution, the value of the expression increases, and when the numbers for x become smaller than the solution, the value of the expression decreases.

Activity 2 [Page 1.3]

1. Angelo's pet rabbit weighs twice the weight of Carmen's pet rabbit.

   a. Does Carmen's or Angelo's rabbit weigh the most?

   Answer: Angelo's rabbit

   b. Angelo's pet rabbit weighs 12 pounds. Write an equation that could be used to find the weight, *w*, of Carmen's rabbit. Use the equation to find the weight of Carmen's rabbit. Does your answer seem reasonable? Why or why not?

   Answer: $2w = 12$ *;* $w = 6$ pounds *so Carmen's rabbit weighs 6 pounds, and that seems like a reasonable weight.*

   c. Angelo's pet rabbit weighs 2 pounds less than twice the weight, *x*, of Simon's pet rabbit. Which rabbit weighs the most, Carmen's or Simon's? Explain your thinking.

   Answer: Simon's rabbit weighs more than Carmen's because Angelo's rabbit is less than twice the weight of Simon's rabbit and is twice the weight of Carmen's rabbit.

   d. Create each of the following equations using the file and find the solutions. Which equation seems reasonable to find the weight of Simon's rabbit and gives a reasonable solution?

   i. $12 = 2x + 2$ ii. $12 = 2x - 2$ iii. $x - 12 = 2$ iv. $w - 2 = 12$

   Answer: Solutions: i) 5 lbs; ii) 7 lbs; iii) 14 lbs; and iv) 14 lbs. Angelo's rabbit has to weigh more so iii and iv are not reasonable equations; Equation i is not correct because it added the 2 pounds to twice the weight rather than subtracting and the solution gives a number less than the weight of Carmen's rabbit. Equation ii is correct because it accounts for the 2 pounds less.

2. Decide whether the following statements are true or false. Give an example to support your thinking. (You may want to use the file to find an example.)

   a. Some equations have more than one solution.

   Answer: True, for example, $0x + 10 = 10$*. Examples may vary.*

   b. Some equations do not have any solutions.

   Answer: True, for example, $0x + 10 = 10$*. Examples may vary.*

   c. Some expressions have an infinite number of solutions.

   Answer: False because an expression is not an equation and so does not have a solution. $2x + 4$ *is not supposed to equal any given outcome, so x can have any value. Examples may vary.*

   d. The values of some expressions will always be even numbers.

   Answer: True, for example $2x + 10$ *will be even for any value of x because the product of 2 and any whole number is even and the sum of two even numbers will be even. Examples may vary.*

Activity 3 [Page 1.5]

1. Write an expression that represents each situation. Let x be the age of Sol's brother.
   a. Sol is one year away from being twice as old as his brother.

      Answer: Both $2x-1$ and $2x+1$ satisfy the constraint.

   b. Summer is three years older than Sol's brother.

      Answer: $x+3$ represents Summer's age.

   c. Sol's dad is five times as old as Sol's brother.

      Answer: 5x represents the age of Sol's dad.

   d. Sol's mom is 5 years younger than Sol's dad.

      Answer: $5x-5$ represents the age of Sol's mom.

2. If Sol is 17 years old, use your work in the question above and write equations to help you find the ages of his mom, dad, brother and sister. You may use the file to solve the equations.
   a. Find Sol's brother's age.

      Answer: $2x-1=17$ or $2x+1=17$, so Sol's brother would be either 8 or 9 years old.

   b. Find the ages of Summer, Sol's dad, and Sol's mom.

      Answer: Summer is $x+3$, where $x=8$ or 9 so she would be either 11 or 12 years old. His dad is 5x years old, so his dad is either 40 or 45 years old, and his mom is $5x-5$ so she would be 35 or 40.

3. A ski resort had a 27-inch snow base at the beginning of the week and six days later it had 42 inches of snow. Let x represents the average number of inches of snow per day over a six-day period.
   a. What does 2x represent? 6x?

      Answer: 2x represents the amount of snow in two days. 6x represents the amount of snow that fell in six days.

   b. Which of the equations $6x=42$, $6x+27=42$ and $6x-27=42$ seem reasonable to represent the total amount of snow at the resort at the end of the sixth day? You might want to use the file to help you figure out your answer. Explain your reasoning.

      Answers. For $6x=42$, $x=7$ inches, which is reasonable but ignores the 27-inch base. For $6x-27=42$, $x=\frac{23}{2}$ and you would have over 66 inches of new snow but this was on top of the old snow, so you would have to add the 27, not subtract it to get to 42. The right equation would be $6x+27=42$ because you would have the new snow plus the base to equal the total amount of snow.

c. The six-day report for four other ski resorts is in the chart. Which resort do you think had the greatest average amount of snow over the six days? The least amount? Explain your reasoning.

Resort	Snow base (inches)	Average snowfall per day (inches)	Depth at end of six days (inches)
Ski Run	32		54
DownHill	25		40
Creekside	34		48
Mountain Run	24		32

Answer: The biggest difference between the base and the end amount of snow was for Ski Run with a 22-inch gain, so it has the highest average amount of snow per day, while the smallest difference was 8 inches for Mountain Run, so it would have the least average amount of snow per day.

d. Use the file to help find the solution for the average amount of snow per day at each of the resorts. Explain how you would have to adjust your equations in each case.

Answer: you would change the constant term on the left expression to the new snow base and the right side of the expression to the new depth.

Resort	*Snow base (inches)*	*Average snowfall per day (inches)*	*Depth at end of six days (inches)*
Ski Run	*32*	$3\frac{2}{3}$ *inches*	*54*
DownHill	*25*	$2\frac{1}{2}$ *inches*	*40*
Creekside	*34*	$2\frac{1}{3}$ *inches*	*48*
Mountain Run	*24*	$1\frac{1}{3}$ *inches*	*32*