Exploring Space Through MATH
Applications in Algebra 2

EDUCATOR EDITION

Newton's Cool in the Pool

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will
- analyze temperature-loss data graphically;
- use Newton's Law of Cooling to predict temperature loss;
- solve the Newton's Law of Cooling formula for the exponential constant of cooling *(k)*; and
- determine the time required for tank temperature to equalize with room temperature.

Prerequisites

Students should have prior knowledge of exponential functions and their inverses, written in both exponential and logarithmic form.

Background

This problem is part of a series that applies mathematical principles in NASA's human spaceflight.

Human spaceflight is an important part of NASA's mission. From lunar exploration to the completion of the International Space Station (ISS), NASA has been preparing humans to explore the unknown. The research and innovation required to explore space has led to technological advancements on Earth. Space exploration has brought benefits to medicine, medical care, transportation, public safety, computer technology, and many other areas that enrich our everyday lives.

Exploring space is a complex endeavor, and missions that involve humans require extensive research, precise planning, and preparation. This includes spacewalks, which are critical for current and future missions.

To prepare for spacewalks, astronauts train at NASA's Neutral Buoyancy Laboratory (NBL) – the largest indoor pool in the world, located at the Sonny Carter Training Facility in Houston, Texas. Besides astronaut training and the refinement of spacewalk procedures, NASA also uses the NBL to develop flight procedures and verify hardware compatibility – all of which are necessary to achieve mission success.

Key Concepts
Exponential and logarithmic functions, Newton's Law of Cooling

Problem Duration
50 minutes

Technology
TI-Nspire™ Handhelds, projector w/ movie player

Materials
- *Newton's Cool in the Pool* Student Edition
- TI-Nspire document, *NBL-Newton.tns*
- *Fluid Dynamics – What a Drag!* video

Skills
Solve exponential and logarithmic equations algebraically; identify characteristics of exponential graphs

NCTM Standards
- Algebra
- Measurement
- Data Analysis
- Problem Solving
- Communication
- Connections

The NBL is 202 ft (61 m) long, 102 ft (31 m) wide, and 40 ft (12 m) deep. It is sized to perform two suited test activities simultaneously, and it holds 6.2 million gallons (23.5 million liters) of water. Even at this size, the complete International Space Station, with dimensions of 350 ft (106 m) by 240 ft (73 m), will not fit inside the NBL (see Figure 1).

The water within the NBL is recycled every 19.6 hours. It is automatically monitored and controlled to a temperature of 82°-88° Fahrenheit to minimize the potential effects of hypothermia on support divers. It is also chemically treated to control contaminant growth while minimizing the long-term corrosion effect on training mockups and equipment.
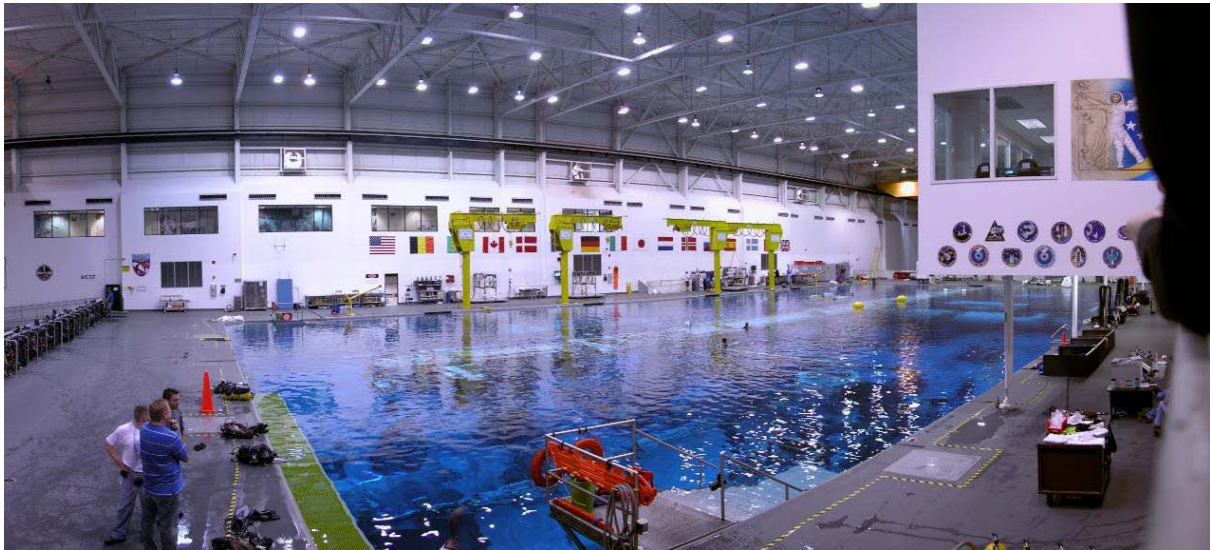

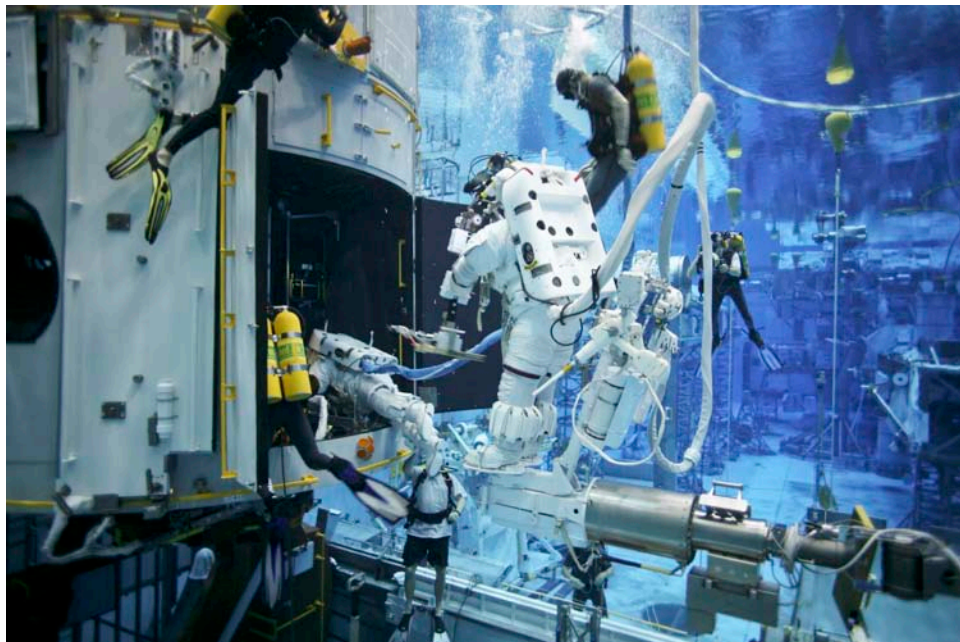*Figure 1: View of entire pool at the Neutral Buoyancy Lab (NBL)*

Figure 2: Astronauts practicing for a spacewalk to repair the Hubble Telescope in the NBL

The NBL allows crewmembers to properly train by experiencing the simulation of a weightless environment in space. With the assistance of divers, suited astronauts are weighted in the pool in order to perform simulated extra-vehicular activities (EVAs) on full mockups of parts of the International Space Station (ISS), the space shuttle cargo bay, and on various payloads.

You may wonder, what is neutral buoyancy and how does it resemble weightlessness? Neutral buoyancy is the equal tendency of an object to sink or float. If an item is made neutrally buoyant through a combination of weights and flotation devices, it will seem to hover under water. In such a state, even a heavy object can be easily manipulated, as is the case in microgravity of space. However, there are two important differences between neutral buoyancy (as achieved in the NBL) and weightlessness. The first is that suited astronauts training in the NBL are not truly weightless. While the suit/astronaut combination is neutrally buoyant, the astronauts can still feel their weight while underwater in their suits. The second is that water drag hinders motion, making some tasks easier to perform in the NBL than in microgravity. While these differences must be recognized by spacewalk trainers, neutral buoyancy is still the best method currently available to train astronauts for spacewalks.

NCTM Principles and Standards

Algebra
- Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology.
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions.
- Use symbolic algebra to represent and explain mathematical relationships.
- Draw reasonable conclusions about a situation being modeled.

Measurement
- Make decisions about units and scales that are appropriate for problem situations involving measurement.

Data Analysis
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

Problem Solving
- Build new mathematical knowledge through problem solving.

Communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections
- Recognize and apply mathematics in contexts outside of mathematics.

Lesson Development

Following are the phases of the 5-E's instructional model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (10 minutes)
- With students in small groups of three to four, ask them to review and discuss the main points of the Background section for several minutes to be sure that they understand the material. Circulate to help facilitate discussion in small groups. Ask if any group needs clarification.
- Play the video, *Fluid Dynamics – What a Drag! (7:13 minutes)*, accessible at the following link: http://www.nasa.gov/audience/foreducators/nasaeclips/search.html?terms=Neutral Buoyancy&category=0010
- Stop the video after three minutes to conserve time. (optional)
- Encourage student discussion of the Background and video, and ask if there are any questions.

2 – Explore (15 minutes)
- Distribute the TI-Nspire file, *NBL-Newton.tns*, to students' handhelds. Most of the information and questions in this activity are embedded in the pages of the TI-Nspire document.
- Ask students to work as a team on pages 1.2–1.7 in the TI-Nspire document.

3 – Explain (10 minutes)
- Have students remain in teams to work on pages 1.8-1.11 in the TI-Nspire document.
- Call on students to give their answers and discuss.

4 – Extend (5 minutes)
- Have students remain in teams to work on pages 1.12- 1.14 in the TI-Nspire document.
- Encourage student discussion and ask if there are any questions.

5 – Evaluate (10 minutes)
- Have students work independently to complete pages 1.15-1.19 in the TI-Nspire document.
- This may be done in class or assigned as homework.

Newton's Cool in the Pool

Solution Key

Throughout this activity, students are given most of the information and questions in the TI-Nspire document, *NBL-Newton.* Some screenshots have been provided throughout the solution key to show what students will be reading on their TI-Nspire handhelds.

Problem

The Neutral Buoyancy Laboratory (NBL) pool is heated to a comfortable temperature of 82°-88° F. Over a long holiday weekend, the pool experienced a power outage, in which the pool's heater stopped working. The power outage occurred over a period of four days, and was not discovered until the divers had a chilling experience when they entered the pool.

Work with your team to predict the temperature change in the NBL pool due to a power failure. Use the formula for Newton's Law of Cooling to determine how long it will take for the water in the pool to reach ambient temperature.
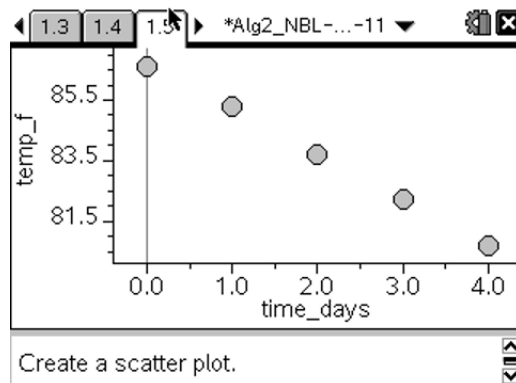
The table on page 1.4 lists temperature data taken in the pool during the power outage. Create a scatter plot on page 1.5.

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings). The formula for Newton's Law is:

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

T(t) = pool temperature, °F, at a given time

T_a= ambient temperature, °F

T_0=initial temperature of the pool, °F

t= time, days

1.7   Solve for *k* in the formula for Newton's Law of Cooling for each daily temperature difference. Use 73° F as the ambient temperature. Round to the nearest thousandth.

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

$$85.3 = 73 + (86.6 - 73)e^{-k(1)}$$

$$85.3 - 73 = (13.6)e^{-k}$$

$$12.3 = (13.6)e^{-k}$$

$$\frac{12.3}{13.6} = e^{-k}$$

$$0.904412 = e^{-k}$$

$$ln(0.904412) = -k$$

$$-0.100 = -k$$

$$T(1),\ k = 0.100$$

Follow the same steps to solve for other T values.

$$T(2),\ k = 0.120$$

$$T(3),\ k = 0.130$$

$$T(4),\ k = 0.142$$

1.8   Notice that the *k*-values are different for each day. Explain why the *k*-value is not constant in this situation.

   Since this is real data, there may be other things that affect pool temperature, such as evaporation and ambient temperature.

1.9   Use the average of the calculated *k*-values to determine the pool temperature on Day 4. Round to the nearest tenth.

$$avg\ k = \frac{0.100 + 0.120 + 0.130 + 0.142}{4} = 0.123$$

$$T(t) = T_a + (T_o - T_a)e^{-kt}$$

$$T(4) = 73 + (86.6 - 73)e^{-(0.123)(4)}$$

$$T(4) = 73 + 13.6e^{-0.492}$$

$$T(4) = 81.3$$

1.10   How close is the calculated temperature to the actual pool temperature on Day 4?

   The calculated temperature is 0.6° F higher than the actual temperature.

1.11   Find the function that represents the data on page 1.4 by substituting your average *k*-value into the Newton's Law equation. Write the function below, and then graph the function together with the scatter plot on page 1.5.

$$T(t) = 73 + 13.6e^{-0.123t}$$

   *Note: To graph the function, click on **analyze**, then **plot function**. F2(x) should appear. Then type in the equation and press **enter**.*
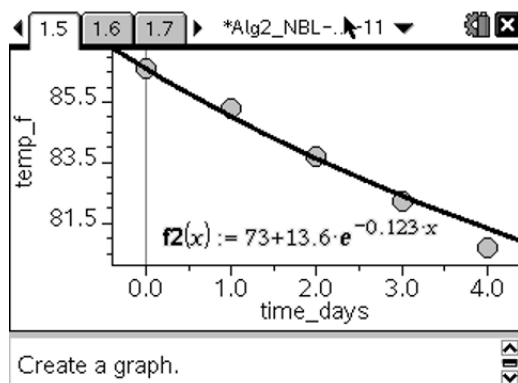
1.12 Determine the number of days it will take the pool to reach a temperature of 74° F. Round to the nearest day. Support your answer graphically on page 1.13.

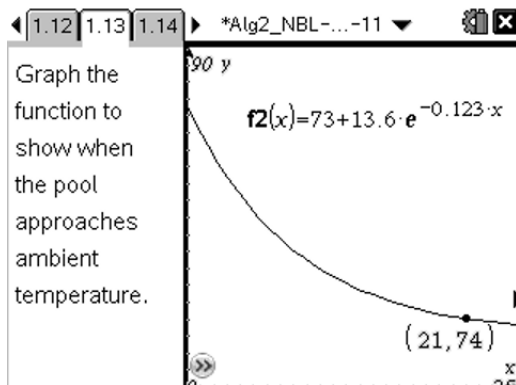$$74 = 73 + (86.6 - 73)e^{-0.123t}$$

$$1 = 13.6e^{-0.123t}$$

$$\frac{1}{13.6} = e^{-0.123t}$$

$$\ln(0.073529) = -0.123t$$

$$\frac{-2.61}{-0.123} = t$$

$$t = 21 \text{ days}$$

1.13 Graph the function to show when the pool approaches ambient temperature.

1.14 What temperature would the pool reach in 10 days? Round to the nearest tenth.

$$T(10) = 73 + 13.6e^{-0.123(10)}$$

$$T(10) = 77.0° F$$

Use the function found on page 1.11 to answer the following questions.

1.16 Is the function increasing or decreasing? How can this be determined from the function rule?

The function is decreasing due to the coefficient of the exponent being negative.

1.17 Is the function asymptotic? Explain.

Yes. The horizontal asymptote is 73° F. This is the lowest temperature that the pool would reach.

1.18   What is the range of the function?

The range of the function is 73<y<∞ or (73, ∞)

1.19   What is the range of the data?

The range of the data is 80.7≤y≤86.6 or [80.7, 86.6]

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

NASA Expert

Kurt Otten – Sonny Carter Training Facility, Neutral Buoyancy Laboratory, NASA Johnson Space Center, Houston, Texas

Mathematics Educator

Suzanne Moyers – C.E. Byrd High School, Caddo Parish Schools, Louisiana