Timekeepers Inc. produces and sells watches. The Company's profit, P, in thousands of dollars, changes based on the number of watches, w, they produce per month.

The rate of change of their profit from producing w watches is modelled by

$$\frac{dP}{dw} = -1.2w + 30, \quad w \geq 0$$

The company makes a profit of 70 (thousand dollars) when they produce 20 watches.

(a) Find an expression for P in terms of w. [5 marks]

(b) The company regularly increases the number of watches it produces. Describe how their profit changes if they increase production to over 25 watches per month and up to 35 watches per month. Justify your answer. [2 marks]

Mark scheme:

(a) Evidence of integration (M1)

$P(w) = -0.6w^2 + 30w\ (+c)$ A1 A1

$70 = -0.6(20)^2 + 30(20) + c$ (M1)

Note: Award ***M1*** for correct substitution of $w = 20$ and $P = 10$. A constant of integration must be seen (can be implied by a correct answer).

$c = -290$

$P(w) = -0.6w^2 + 30w - 290$ A1

[5 marks]

(b) Profit will decrease (with each new watch produced) A1

Either

Because the profit function is decreasing/ the gradient is negative/ the rate of change of P is negative R1

Or

$\int_{25}^{35} -1.2w + 30\ (dx) = -60$ R1

Or

Evidence of finding $P(25) = 85$ and $P(35) = 25$ R1

Note: Award at most **R1A0** if $P(25)$ or $P(35)$ or both have incorrect values.

[2 marks]