**To Infinity and Beyond!**
Student Activity

Name _____

Class _____

The limit of a function is the behavior the output value approaches as the input value approaches a particular value. Limits are also examined when the output value keeps increasing or decreasing without bound. In this case the notation is $\lim_{x \to \infty}(\ )$, as is read as "the limit as the input value approaches infinity." The limit exists when the function moves toward a single output value and fails to exist when it does not move toward one value.

```
NORMAL FLOAT AUTO REAL RADIAN MP        Y₁
```

```
X=0                    Y=1000
```
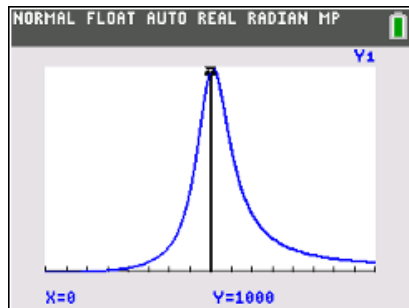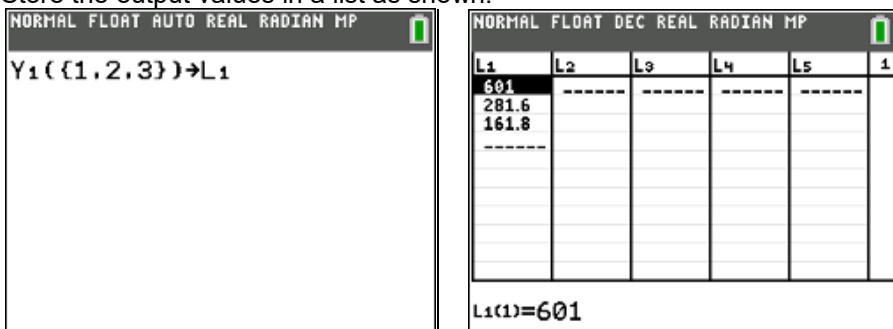

This activity has three parts. First, you will examine, graphically and numerically, the behavior of functions as the input approaches infinity. Next, you will examine graphically limits that do not exist because of continued chaotic output behavior as the input values continue to approach a particular value. Finally, you will examine a variety of limit problems.

Problem 1

Input this function into **Y₁** in the **Y =** editor: $f(x) = \dfrac{2x^2 + 200x + 1000}{x^2 + 1}$

(a) Record the function values for the inputs {1, 2, 3}. _____

(b) Store the output values in a list as shown:

```
NORMAL FLOAT AUTO REAL RADIAN MP

Y₁({1,2,3})→L₁
```

```
NORMAL FLOAT DEC REAL RADIAN MP

L₁      L₂      L₃      L₄      L₅      1
601     ------  ------  ------  ------
281.6
161.8
------



L₁(1)=601
```

(c) Repeat the process for the input values {100, 200, 300}. Store the output values in **L₂**, and record the values.

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(d) Repeat the process for the input values {1000, 2000, 3000}. Store the output values in **L₃**, and record the values.

_____

1

(e) With a classmate, look at the values in all three sets and draw a conclusion regarding the behavior of the function.

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(f) To confirm your conclusion, try the input values {10^{10}, 10^{15}, 10^{20}}, and record your results.

_____

(g) With a classmate, discuss if the function actually reaches the exact value of the limit. Share with the class if you think this is a reasonable result.

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(h) Now let's look at the behavior graphically. Take several minutes exploring what viewing windows you would need to see the behavior from the input values listed in parts a, c, d, and f. Compare your windows on the handheld with a classmate and discuss what you notice.

(i) Estimate: $\lim\limits_{x\to\infty} \dfrac{2x^2+200x+1000}{x^2+1}$ _____

(j) With a classmate, investigate and draw a conclusion of the following limit:

$$\lim\limits_{x\to-\infty} \dfrac{2x^2+200x+1000}{x^2+1}$$

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Problem 2

Now investigate the limit $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ as you did in **Problem 1**. Fill in the following table given the following input values:

X	L₁	X	L₂	X	L₃	X	L₄
1		100		1000		10^{10}	
2		200		2000		10^{15}	
3		300		3000		10^{20}	

(a) With a classmate, discuss what you notice as the input values approach infinity.

_____

_____

(b) Given the following viewing window, graph the function and use the **trace** button to help describe what you see.

```
NORMAL FLOAT DEC REAL RADIAN MP
FREE TRACE VALUES
WINDOW
 Xmin=-100000000
 Xmax=1000000000000
 Xscl=0
 Ymin=0
 Ymax=4
 Yscl=1
 Xres=1
 ΔX=3788257575.7576
 TraceStep=7576515151.5152
```

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_____

(c) State and explain the limit: $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

_____

_____

Problem 3

Before you practice limits on your own, let's examine the behavior of one more function: $f(x) = \sin\left(\frac{1}{x}\right)$

Use the following window, make sure your handheld is in radian measure and graph $f(x)$. Please set your **Zoom Factors** by pressing **zoom**, **memory**, **4: SetFactors**. Make sure **XFact = 4** and **YFact = 1**.

Discuss with a classmate what you notice about the function as the input values approach 0 and describe your results of the limit: $\lim\limits_{x \to \infty} \sin\left(\frac{1}{x}\right)$

_____

_____

Practicing Limits

Evaluate the limit for each expression:

1. $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x}$ _____

2. $\lim\limits_{x \to \infty} \dfrac{x-2}{2x+5}$ _____

3. $\lim\limits_{x \to 0} \dfrac{|x|}{x}$ _____

4. $\lim\limits_{x \to 0} \dfrac{\sin(5x)}{4x}$ _____

5. $\lim\limits_{x \to -\infty} \dfrac{4x-1}{\sqrt{x^2+2}}$ _____

6. $\lim\limits_{x \to 1} \dfrac{x^3-1}{x-1}$ _____

7. $\lim\limits_{x \to 2} \left(\dfrac{1}{x-2} - \dfrac{4}{x^2-4}\right)$ _____
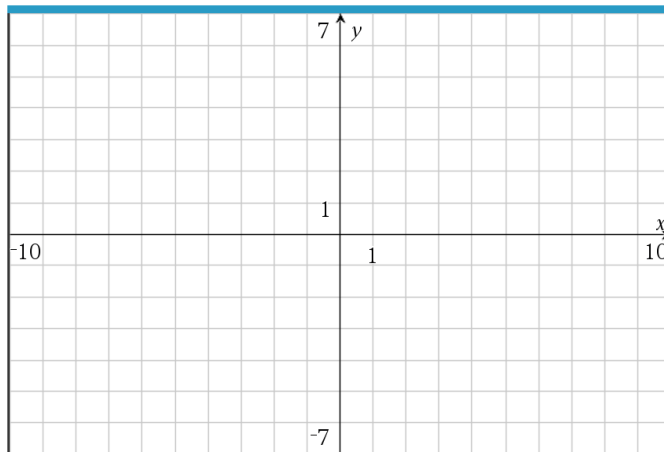
Further IB Applications

1. (a) Sketch the graph $y = \frac{x^2-4}{x-2}, x \neq 2$.
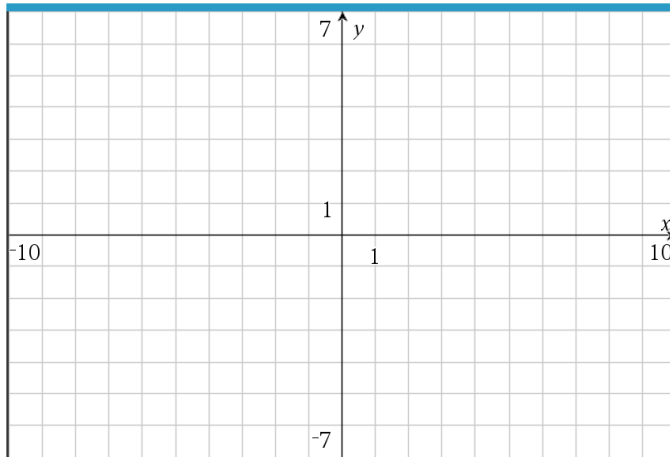
(b) Find $\lim\limits_{x \to 2} \frac{x^2-4}{x-2}$ numerically.

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2. (a) Consider the function $f(x) = \frac{3x}{x-2}, x \neq 2$. Sketch the function.

(b) Evaluate the limit: $\lim\limits_{x \to \infty} \frac{3x}{x-2}$

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