Discrimination of Parabolas

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Professional Development from Texas Instruments

Answers - ACMMM010

7 8 9 **10** **11** 12

TI-Nspire™ | Activity | Student | 30 min

Introduction

This investigation explores a special number that is explicitly used to determine a single characteristic of a quadratic function or parabola. Your task is to determine the role of this special number. In the first part of the investigation you will be provided with parabolas that have slightly restricted movement. On the basis of the information you collect about these parabolas you will be required to develop conjectures. These conjectures will later be refined as more evidence is formed. While your conjecture should be thoroughly tested at each stage, it will most likely be modified as more evidence is revealed.

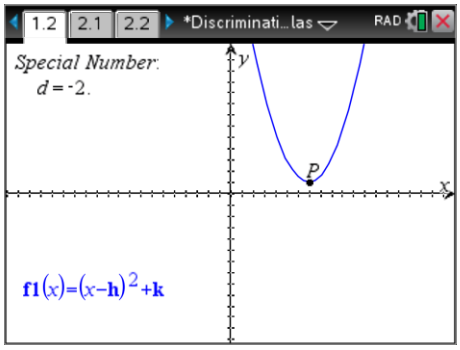
Equipment

- TI-Nspire Calculator
- TI-Nspire file: Discrimination of Parabolas

Investigation

Conjecture – Opinion formed on the basis of incomplete information

Open the TI-Nspire file: Discrimination of Parabolas and navigate to Page 1.2. The 'special number' is displayed in the top left corner of the screen. Point P is located at the *vertex* or *turning point* of the parabola. This point can be moved around the Cartesian plane.



Move the mouse over point P. When the mouse turns into a small hand, click and hold (or press Ctrl then click) to grab point P.

Drag point P around the Cartesian plane and observe the value of the special number: *d*.

Question: 1.

Write down any initial observations you can make about the parabola and the value of *d*.

Answers will vary, samples answers are provided here:

- Always even
- Negative when the parabola is above the x axis and positive when it is below.

Remember students are making conjectures based only on the information they have available.

Conjectures that contain information inconsistent with the interactive diagram however should not be marked correct.

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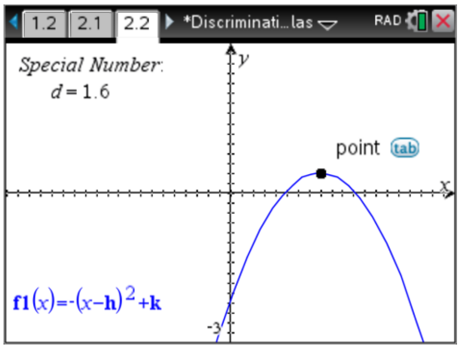
Navigate to page 2.2. The parabola on this page has been inverted. Drag Point P around once again.

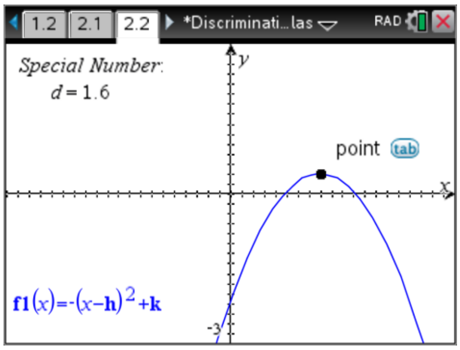


Question: 2.

Review any conjectures made in Question 1 and re-write as necessary to incorporate information relating to the special number *d* and the parabolas on page 1.2 and 2.2.

- *d can take on any value,* $d \in R$ *(Contrary to initial conjecture)*
- Negative when the parabola is completely below the x axis and positive when it is above, the reverse of the previous conjecture … due to the inversion of the parabola.

Navigate to page 3.2. The parabola on this page is completely flexible.



To translate the parabola, *grab* the vertex as before.

To dilate the parabola, move the mouse over another region (other than the vertex) until a double sided arrow appears:

Once the double sided arrow appears press Ctrl + Click to grab the parabola and dilate it.

Question: 3.

Review any conjectures made in Questions 1 & 2 and re-write as necessary to incorporate information relating to the special number *d* and the general parabola on page 3.2.

- *If d > 0 the parabola crosses the x axis.*
- *If d < 0 the parabola does not cross the x axis.*
- *If d = 0 the parabola 'touches' the x axis in just one location. (May not be realised by many students.)*

There are many conjectures in mathematics, theories that are yet to be proved; a simple example is that every even number (greater than two) can be represented as the sum of two prime numbers. This theory remains as a conjecture because it has yet to be proved. Exploring lots, even millions of different even numbers is not sufficient proof as an infinite number of even numbers exist, so it is not possible to test them all. So far many parabolas have been explored developing your conjectures, but they remain conjectures until 'all' parabolas have been explored.

Question: 4. (Extension)

The *special* number is computed as: $b^2 - 4ac$ where: $y = ax^2 + bx + c$ is used to represent any parabola. Prove that your conjecture works for all parabolas.

Conjecture: The *special* number determines if the parabola crosses the x axis. In order to cross the x axis it is a requirement that y = 0. Using algebra (or CAS) … $solve(ax^2+bx+c=0,x)$ produces the result: $x=\frac{\sqrt{b^2-4ac}-b}{2a}$ or $x=\frac{-\left(\sqrt{b^2-4ac}+b\right)}{2a}$. The question provides a clue to look for the *special* number: b^2-4ac. Students should note that if this special number (discriminant) is less than zero it will result in a 'non-real' result, implying that there are no x intercepts. When this value is greater than zero two solutions will be provided (two x intercepts) and that if this value is 0, both of these expressions are equivalent… only one x intercept.

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