How dense are we?

Teachers Notes and Answers

TI-Nspire™   Activity   Student   30 min

Introduction

Australia celebrated a population milestone on August 7th 2018 when it was estimated, based on census data, that our population reached 25 million. The same calculations estimate Australia will have a population of 30 million by 2030. Where will they all fit? Almost 70% of Australians reside in capital cities, the majority of them in Melbourne or Sydney. Based on these figures and trends it seems logical that these cities will continue to grow. It would make more sense to encourage growth in areas that are less dense.

This activity explores the density of Australia's states and territories, capital cities and regional areas. It must be remembered however that exact figures are relatively meaningless. It is essentially impossible to state exactly how many people reside in the country at any moment in time. Furthermore by the time you complete this task that figure will have changed. Scientific notation is therefore a great way to communicate the general magnitude. Similarly the exact area of a state is based on calculations by cartographers. The exact area of a state would technically change when the tide is out! When determining population density, consideration must also be given to inhabitability.
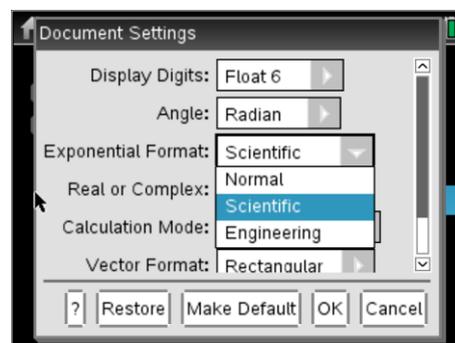
Exploration

Start a new TI-Nspire document. Change the document settings so that the Exponential Format is set to Scientific.

Home > **Settings** > **Document Settings**

Also check that the Display Digits is set to Float 6.

Use the 'OK' option rather than Make Default so that the changes apply only to this document.

Scientific notation is often used when the numbers become very big or very small. However, there is a difference between mathematical scientific notation and calculator scientific notation. You will learn how it works after completing this task.

Tip

This activity includes the use of notation reserved for electronic devices where "E" is used to reference "× 10". This reserved notation should **not** be used in written mathematical calculations or communications. The Extension question at the end of this investigation addresses critical differences in the way associated calculations are performed.

Authors: Bozenna Graham & Peter Fox

To enter a number in standard notation we use the EE button located to the left of the letter "A".

In a calculator application type in:

1 E 6   (Pressing EE produces the exponential E)

Notice that the calculator will change the display after the number is entered. The left hand side of the screen displays the entire number but the right hand side displays the number in scientific notation.
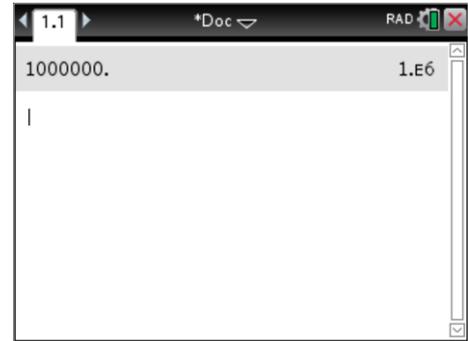
Question 1:

Enter each of the following numbers below and write down the output generated by the calculator:

a.   52E3 (5.2E4)

b.   2018E6 (2.018E9)

c.   0.00015E3 (1.5E-1)

d.   0.001 (1E-3)

e.   50E-3   (5E-2)

f.   0.075E-5 (7.5E-6)

In each case explain any changes to the exponent.

The exponents are increased / decreased such that only a single digit precedes the decimal place.

Question 2:

Calculate each of the following providing answers in exponent form:

a. $3.2 \times 10^5 \times 2 \times 10^5$

6.4×10^{10}

b. $1.9 \times 10^3 \times 3 \times 10^4$

5.7×10^7

c. $6.1 \times 10^4 \times 5.2 \times 10^8$

3.172×10^{13}

d. $\dfrac{3.2 \times 10^8}{1.6 \times 10^3}$

2×10^5

e. $\left(2 \times 10^5\right)^3$

8×10^{15}

f. $\dfrac{1}{2 \times 10^4}$

5×10^{-5}

In each case explain any changes to the exponent.

In most cases the exponent follows the corresponding index law where the bases are the same. Exceptions occur with the digit term results in a numerical value where there is no longer a single leading digit preceding the decimal place. Questions 2(a) and 2(b) show the simple addition of the exponents for the common base whereas 2(c) includes a digit multiplication of: $6.1 \times 5.2 = 31.72$. This can be written as: 3.172×10 so the original calculation becomes: $3.172 \times 10 \times 10^4 \times 10^8 = 3.172 \times 10^{13}$.

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The table below shows the population in each state in Australia at the end of December 2017 (ABS Data[1]) and the area of each state in square kilometres.

State	Population	Area (km^2)
New South Wales	7.915×10^6 (32.0%)	8.006×10^5 (10.0%)
Victoria	6.386×10^6 (25.8%)	2.274×10^5 (2.96%)
Queensland	4.965×10^6 (20.0%)	1.731×10^6 (22.5%)
South Australia	1.728×10^6 (6.98%)	9.835×10^5 (12.8%)
Western Australia	2.585×10^6 (10.4%)	2.530×10^6 (22.5%)
Tasmania	5.247×10^5 (2.12%)	6.840×10^4 (0.89%)
Northern Territory	2.467×10^5 (1.00%)	1.349×10^6 (17.5%)
Australian Capital Territory	4.159×10^5 (1.67%)	2.358×10^3 (0.03%)

Question 3:

Express Victoria's population and land area in basic numerals. 6,386,000

Question 4:

Calculate Australia's total population at the end of 2017 (According to the data). 2.477×10^7

Question 5:

Calculate Australia's total land area. 7.692×10^6

Question 6:

Calculate the percentage of Australia's area for each state. (See table)

Question 7:

Calculate the percentage of Australia's population for each state.   (See table)

Question 8:

Calculate the population density (population ÷ area) for each state. (Units: People per square kilometre)

NSW	9.88	VIC	28.1
QLD	2.87	SA	1.76
WA	1.02	TAS	7.67
NT	0.18	ACT	176

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Question 9:

Japan has a population of approximately 127 million people and a land area 377,972km². Determine the population density of Japan.   336 people per square kilometre.

Question 10:

If every Australian moved to Tasmania, would the population density in Tasmania be more or less than that for Japan?   362 people per square kilometre, therefore slightly higher population density than Japan.

Extension

Explain why $6 \times 10^8 \div 3 \times 10^4$ produces a different answer to $\dfrac{6 \times 10^8}{3 \times 10^4}$.

Compare the answers for the two calculations above with: 6ᴇ8 ÷ 3ᴇ4

In the first calculation, the order of operations produces: $(6 \times 10^8 \div 3) \times 10^4 = 2 \times 10^{12}$

The second calculation is the most likely intention producing the result: 2×10^4 which is the same as that for the scientific notation. The scientific notation therefore represents a fast way to perform such calculations, but be wary, it may be at the expense of understanding for some students.

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