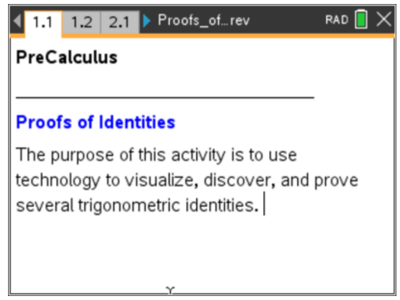
Proofs of Identities
Student Activity

Name ______________

Class ______________

Open the TI-Nspire document *Proofs_of_Identities.tns.*



An identity is an equation that is true for all values of the variables for which both sides of the equation are defined. In this activity, you will discover some trigonometric identities by manipulating the graphs of trigonometric functions. You will then use your algebraic skills and your knowledge of trigonometric representations to prove these identities.

Move to page 1.2.

This page shows the graphs of two identical functions: f1(x) = 1 and f2(x) = 1. Throughout this activity, the graph of f1(x) is shown as a thin, solid line and the graph of f2(x) is shown as a thicker dashed line.

1. Press ctrl G to access the function entry line. Press the up arrow (▲) to access the equation for f1(x) and change it to csc(x). Change the equation for f2(x) to $\dfrac{1}{\sin(x)}$.

   What do you notice about the two graphs? Why is your conclusion true?

2. Complete the information for the other "Reciprocal Identities". To do so, you might want to change the equations for f1(x) and f2(x) to help you visualize the identities.
   a. $\sec(x) =$

   b. $\cot(x) =$

   c. $\sin(x) =$

   d. $\cos(x) =$

   e. $\tan(x) =$

3. Why are these statements called Reciprocal Identities?

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Move to page 2.1.

Click on the slider to transform $f2(x) = a \cdot \sin(x)$ into $f1(x) = \sin(-x)$.

4. How does changing the parameter, *a*, affect the graph of $f(x) = a \cdot \sin x$?

5. Write the equation for $f2(x)$ with a specific value for *a* such that $f2(x) = f1(x)$.

6. Using the information above, fill in the blank:

$$\sin(-x) = \_\_\_\_\_ \sin(x).$$

Move to page 2.2.

This page shows the unit circle and the definitions for three basic trigonometric functions in terms of *x*, *y,* and *r*. The segment labeled *r* has been reflected over the x-axis. Drag point *P,* and observe how the coordinates of point P and its image point P' change.

7. Consider the relationship between the coordinates of P and the coordinates of P'. Write the coordinates of P and the image point, P', in terms of *x* and *y*.

8. Using the coordinates of the image point P' and the definitions of the basic trigonometric functions shown on the screen, write an expression for sin($-\theta$) in terms of sin(θ).

Move to page 2.3.

Click on the slider to transform $f4(x) = a \cdot \cos(x)$ into $f3(x) = \cos(-x)$.

9. How does changing the parameter, *a*, affect the graph of $f(x) = a \cdot \cos x$?

10. Follow the directions in questions 5 and 6, using the cosine function instead of the sine function. Write the equation for $f4(x)$ in the space below.

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Move back to page 2.2.

11. Using the coordinates of the image point P' and the definitions of the basic trigonometric functions shown on the screen, write an expression for cos(–θ) in terms of cos(θ).

Move to page 2.4.

Click on the slider to transform $f6(x) = a \cdot \tan(x)$ into $f5(x) = \tan(-x)$.

12. Follow the directions in questions 5 and 6, using the tangent function instead of the sine function. Write the equation for $f6\left(x\right)$ in the space below.

13. Use the information you obtained about $\sin\left(-\theta\right)$ and $\cos\left(-\theta\right)$ in questions 8 and 11 to write an expression for tan(–θ) in terms of tan(θ).

14. Use the information from above to fill in the information below for the "Negative Angle Identities."
    a. $\sin\left(-\theta\right) =$

    b. $\cos\left(-\theta\right) =$

    c. $\tan\left(-\theta\right) =$

Move to page 3.1.

Click on the slider to transform $f2(x) = \cos\left(\dfrac{\pi}{a} - x\right)$ into $f1(x) = \sin(x)$.

15. How does changing the parameter, *a*, affect the graph of $f(x)=\cos\left(\frac{\pi}{a}-x\right)$?

16. Use the equation for $f2(x)$ to fill in the blank:

$\sin(x)=$______________.

Move to page 3.2.

This page shows a right triangle with acute angles α and β labeled. Press the arrow (▲) to see the image of the triangle after a copy of the right triangle is placed so that the two triangles form a rectangle. Press the arrow (▲) again to see the measures of two additional angles labeled as α and β.

17. Why are the two angles in the image labeled as they are?

18. What is the relationship between α and β? Explain your answer.

Move to page 3.3.

19. a. Click and drag vertex *A* and vertex *B*. What do you notice about $\sin(\beta)$ and $\cos(\alpha)$? Write an equation to express $\sin(\beta)$ in terms of α.

b. Substitute $\frac{\pi}{2}-\beta$ for α to re-write the equation above in terms of β.

20. We see from questions 17 - 19 that the sine of an acute angle is equal to the cosine of its complement. The sine and cosine functions are called *cofunctions*. The tangent and cotangent functions are also *cofunctions*. Use this information to fill in the blanks below for the "Cofunction

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Identities."

a. $\sin(\theta) = \cos($ ____________ $)$

b. $\cos(\theta) = \sin($ ____________ $)$

c. $\tan(\theta) =$ ________________

Move to pages 3.4 and 3.5.

Use these pages to support the identities you wrote in question 20.

Move to page 4.2.

Move the sliders to change the values of the two parameters in $f2(x)$ to transform the graph of $f2(x) = a \cdot (\sec(x))^2 + b$ into $f1(x) = (\tan(x))^2$.

21. Write the equation for $f2(x)$ with specific values for *a* and *b* such that $f2(x) = f1(x)$.

Move to page 4.3.

This page shows a right triangle and the definitions of three basic trigonometric functions in terms of the lengths of the sides.

22. Divide both sides of the Pythagorean Formula by a^2 as shown, and simplify the result. Substitute the appropriate trigonometric functions for the ratios $\frac{b}{a}$ and $\frac{c}{a}$, and rewrite the equation below.

23. Repeat the process, dividing by b^2 and then by c^2 to yield two additional identities. Fill in the information below for the "Pythagorean Identities."

    a. ______________ = $\sec^2(x)$

    b. __________+ $\cot^2(x)$ = ___________

    c. $\sin^2(x)$ + _________= __________

Move to page 4.4.

Use this page to visually support the identities you derived in question 23. Press ctrl G to access the function entry line. Press the up arrow (▲) to access f3(x). Enter the left half of the equation from #23b as f3(x) and the right half of the equation as f4(x). Repeat the process for the equation for #23c.

24. a. Using the Pythagorean Identity, $\sin^2\theta+\cos^2\theta=1$, divide all three terms by $\sin^2\theta$ and simplify the fractions to write another Pythagorean Identity.

    b. Use the identity $\sin^2\theta+\cos^2\theta=1$ to divide all three terms by $\cos^2\theta$, and simplify the fractions to write another Pythagorean Identity.