Which Garage is Better?

IB Aligned

Math Objectives

- Students will find the solution to a system of linear equations by hand.
- Students will use the TI-84 Plus CE to graph a linear system in two variables and find the intersection point(s).
- Students will create and graph linear piecewise functions.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- linear system
- piecewise function

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 2 Functions:

  2.4b Finding the point of intersection of two curves or lines using technology.
- As a result, students will:

  Apply this information to real world situations.

Teacher Preparation and Notes

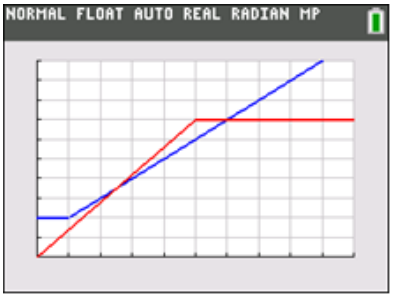
- This activity uses the TI-84 family handhelds as an aid to the problems, specifically graphing and analyzing piecewise and linear functions.
- This activity serves as a good lesson to culminate solving systems using technology.

Activity Materials

- Compatible TI Technologies:

TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Which_Garage_is_Better_Student-84.pdf
Which_Garage_is_Better_Student-84.doc

Tech Tip: Before beginning the activity, it may be beneficial to discuss with students how to graph a piecewise function on the handheld, especially if it is not a TI-84 Plus CE with an updated operating system.

Problem 1 – A Parking Garage

The cost schedules for two different parking garages are below. The maximum stay is 24 hours.

Blue Street Garage	
Length of time in the garage	**Cost**
1 hour or less	A flat fee of $10.
More than 1 hour	$10 for the first hour plus an additional fee of $5 for every hour after your first hour in the garage.
Red Street Garage	
Length of time in the garage	**Cost**
5 hours or less	$7 per hour
More than 5 hours	A flat fee of $35

1. Complete the table.

Length of time in the garage (hours)	**Blue Street Garage total cost (dollars)**	**Red Street Garage total cost (dollars)**
0	**10**	**0**
1	**10**	**7**
2	**15**	**14**
3	**20**	**21**
4	**25**	**28**
5	**30**	**35**
6	**35**	**35**
7	**40**	**35**
8	**45**	**35**
9	**50**	**35**

Answer: Students may find it easier to compete the table for the Red Street Garage first. Using the slope, students might build the table for the Blue Street Garage by adding 5 to the previous row of the table.

2. Joe parked in the Blue Street Garage and Flo parked in the Red Street Garage for the same length of time. After they checked out and paid, they asked each other which garage was cheaper, only to discover they paid the same amount for their stay.

   Using the table above, answer the following questions:

   (a) Find the length of time each might have parked in the garage. Find all possible answers.

   Answer: Each could have stayed 6 hours and paid $35, but that is not the only possible answer. Red surpasses Blue between 2 and 3 hours. Using proportional reasoning, for each half hour the cost of the Blue Street Garage increases by ½ of 5 or $2.50, and the cost of the Red Street Garage increases by ½ of 7 or $3.50.

   (b) Find what their fee would have been. Find all possible answers.

   Answer: Each could have stayed only 2.5 hours and paid $17.50. This will be more apparent with the graph.

3. Write the equations of piecewise functions that model the cost of staying in each of the garages.

Blue Street Garage: $B(x)$ =

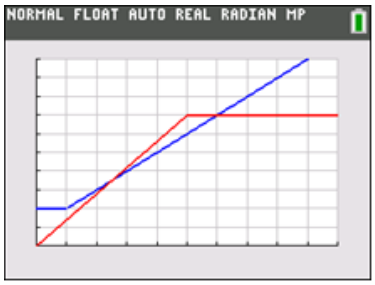
Answer: $B(x) = \begin{cases} 10, & x \leq 1 \\ 5(x-1)+10, & x > 1 \end{cases}$

Red Street Garage: $R(x)$ =

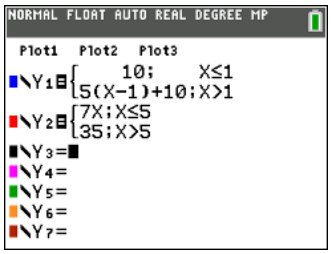
Answer: $R(x) = \begin{cases} 7x, & x \leq 5 \\ 35, & x > 5 \end{cases}$

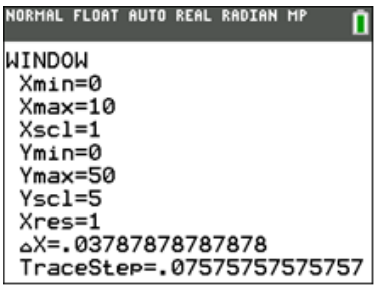
Teacher Tip: If your students are new to writing piecewise functions, a scaffolding idea is to give them the structure and have them fill in the rest of it. For example, give them the first line and let them figure out the second part.

4. To graph a piecewise function on your TI-84 Plus CE, press **y =**, **math**, **B: piecewise(**, select 2 pieces and select **OK**. Enter your equations and inequalities into the four empty boxes. You will repeat this for the Red Street Garage as well. Your graph should look like the one to the right using the viewing window shown.



Answer:





Teacher Tip: Use the TI-Smartview™ CE Emulator Software for the TI-84 Plus Family to show Keypress History.

5. Find when the costs for using each garage will be equal to each other. Write down the equations you would set equal to each other to find when the costs are equal. Solve these equations below. Verify your answer by examining the intersection on the handheld. Press **2nd**, **trace**, **5:intersect**.

Equation 1: _____

Answer: $7x = 10 + 5(x - 1)$

$7x = 10 + 5x - 5$

$2x = 5$

$x = 2.5$

Equation 2: _____

Answer: $10 + 5(x - 1) = 35$

$5(x - 1) = 25$

$x - 1 = 5$

$x = 6$

6. Use the table, formula, or graph to answer the following. Press **2nd**, **graph** for the **table**.

a. Find which garage costs less for a short stay. For example, you enter the garage, park, realize you forgot your wallet, and end up having to leave only 15 minutes later.

Answer: Students may need to be reminded of units. They can evaluate each function at x = 0.25 hours or just use the graph. The Red Street Garage is cheaper since the blue Street Garage is $10 and the Red Street Garage is $1.75.

b. Suppose after a 2.5-hour movie you decide to go out to a restaurant and stay an additional 2 hours. Find which garage will cost less. State how much less.

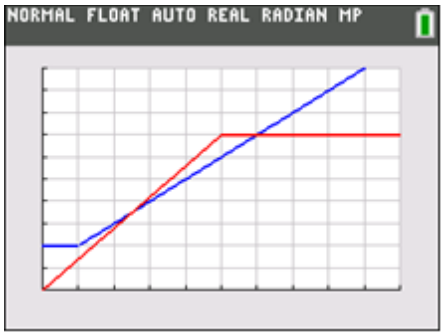
Answer: If you stay 4.5 hours, the Blue Street Garage ($25 + $2.50 = $27.50) is cheaper than the Red Street Garage (4.5 x $7 = $31.50) by $4. You can also use the table feature on the handheld.

c. Suppose you needed to park your car for 12 hours in the garage. State which garage will cost less. State how much less.

Answer: If you stay 12 hours, the Red Street Garage ($35) is cheaper than the Blue Street Garage ($10 + $55 = $65) by $30. You can also use the table feature on the handheld.

7. Use the graph to solve when $B(x) < R(x)$. Interpret the solution in real-world practical terms.

Answer: In terms of the context of parking garages, the Blue Street Garage is cheaper than the Red Street Garage if you stay more than 2.5 hours but less than 6 hours.



8. Over the length of a day, find what duration the Blue Street Garage is better. Find what duration the Red Street Garage is better.

Answer: The Blue Street Garage is better for a duration of 3.5 hours (when you stay more than 2.5 hours but less than 6 hours). The Red Street Garage is better for the remaining 20.5 hours of the day (for hours 0 to 2.5 and 6 to 24).

Problem 2 – Music Sales

In recent years, the numbers of CDs sold in the United States has declined while digital music has become the new method for purchasing music.

The table below shows data of the sales, in millions, of CDs, digital albums (**DA**), and individual songs (**IS**) for the first three months of the year.

Year	CD	DA	IS
2006	112	119	24.2
2007	89	99	28.8
2007 – 2006	-23	-20	4.6

9. Describe what the value of -23 (under CD) represents.

Answer: The difference between the sales data for CDs and, since there is only a difference of one year, it is also equal to the slope of the line.

10. Discuss with a classmate and write down why you think it is negative.

Answer: Because there was a decrease in sales of CDs from 2006 to 2007.

11. Discuss with a classmate how the result in the final column (IS) is different from the other two (CD and DA) and explain why.

Answer: The result is positive because the individual song sales went up for the same time period.

12. Use the data in the table above to find the equation for each of the three lines in either slope-intercept form or point-slope form and write them in the spaces provided.

- CDs _____

Answer: $y = -23x + 46250 \ or \ (y - 112) = -23(x - 2006)$

- Digital Albums _____

Answer: $y = -20x + 40239 \ or \ (y - 119) = -20(x - 2006)$

- Individual Songs _____

Answer: $y = 4.6x - 9203.4 \ or \ (y - 24.2) = 4.6(x - 2006)$

Use the lines to find and record the coordinates of the three intersection points.

CDs and Digital Albums (_____ , _____)

Answer: $(2003.67, 166)$

CDs and Individual Songs (_____ , _____)

Answer: $(2009.18, 38.8)$

Digital Albums and Individual Songs (_____ , _____)

Answer: $(2009.85, 41.9)$

13. Find when the sales of digital albums over takes the CDs.

Answer: The later part of 2003.

14. Find when the graph projects that the sales of individual songs overtakes CDs.

Answer: The beginning part of 2009.

15. Find when the graph projects that the sales of individual songs overtakes digital albums.

Answer: The later part of 2009.

16. As time goes on according to the graph, it indicates the CD sales becoming zero. Discuss with a classmate if you think this is possible. Explain why or why not.

Answer: The sales of CDs will continue to go down but probably will not go away, until the industry decides to stop making CDs.

IB Further Extension

To further instruction, finding where two functions are equal should not be limited to linear functions. In this extension, you will be exploring where two exponential functions are equal.

Luca purchases a new bike for himself at a cost of $355. He also purchases a professional racing bike for his sister Christine for $1815. Luca's bike will depreciate in value 5% per year, while Christine's will depreciated at a rate of 12% per year. Luca and Christine's bikes will have the same value *p* years after they were purchased.

(a) Estimate the value of Luca's bike after 6 years.

Answer: Students will first have to write the given information as an exponential equation.
Luca: $L(t) = 355(1 - 0.05)^t$
$L(6) = 355(0.95)^6$
$L(6) = 260.958 \approx \$261$

(b) Using your handheld, find *p*.

Answer: Christine: $C(t) = 1815(1 - 0.12)^t$
$C(t) = 1815(0.88)^t$

Set the exponential equations equal to solve for *p.*
$355(0.95)^p = 1815(0.88)^p$

Finding their point of intersection using the graphs on their handheld,
$(21.3185, 118.94)$
After approximately $p = 21.3$ years, their bikes will have an approximate value of $118.94.

(c) Explain why or why not the answer to part (b) is valid.

Answer: Depreciation rates are not constant from year to year, especially over long periods of time.

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