Math Objectives

- Students will identify the standard deviation as a measure of spread about the mean of a distribution.
- Students will recognize that the size of the standard deviation depends on the spread of the distribution.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

- deviation
- dot plot
- mean
- range
- standard deviation

About the Lesson

- This lesson is intended as an introductory activity to the concept of standard deviation.
- As a result, students will:
  - Manipulate a dot plot to obtain specified standard deviations.
  - Examine different distributions with the same mean and range and note how the standard deviation changes.
  - Gain a basic understanding of what standard deviation is measuring and how changes in data values affect the standard deviation.

Prerequisite

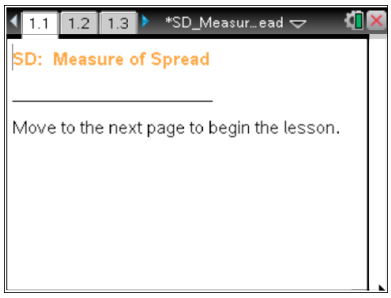
Students should be familiar with the concepts of mean and range.

Related Activities

- SD: How Far is Typical?

TI-Nspire™ Navigator™ System

- Use Screen Capture to show that distributions with the same mean and range can have different standard deviations.
- Use Live Presenter to demonstrate how to find two different distributions that have the same mean and standard deviation.
- Use Quick Poll to assess students' understanding of standard deviation.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab, drag and deselect a point in a dot plot

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

SD_Measure_of_Spread_Student.pdf

SD_Measure_of_Spread_Student.doc

TI-Nspire document

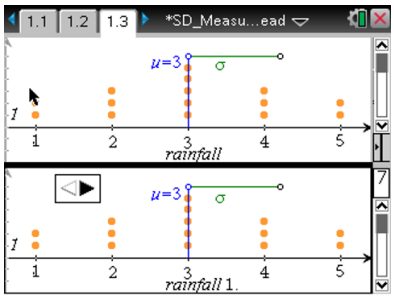
SD_Measure_of_Spread.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: Note that the vertical line on page 1.3 of the .tns file represents the mean of the distribution.

Move to page 1.3.



1. The graphs on page 1.3 display a distribution of hypothetical total rainfall to the nearest inch for a given month in a given year for the 15 states in the region of the United States classified as the South. The mean of the data set is indicated by the vertical line segment and the standard deviation by the horizontal line segment.
   a. Describe the information the mean provides about the amount of rainfall in the South for that month.

Sample Answers: The mean is a measure of center and represents the typical amount of rainfall in inches, 3 inches, in this distribution. This indicates that the typical rainfall for that month for the 15 Southern states is 3 inches. Some students might describe this as an arithmetic average.

Teacher Tip: The federal government sometimes breaks the country into four regions—West, Midwest, Northeast and South—to report data in some categories (such as graduation rates). The South consists of 15 states and the District of Columbia (DC). DC is not included in the set of states in this activity. You might have students explore questions throughout the activity such as what would the standard deviation be if the states had about the same amount of rainfall; what would happen to the standard deviation if one of the states in the South had a year with very little rainfall?

   b. Find the range, and describe the information it provides.

Sample Answers: For the given month the highest amount of rainfall is 5 inches, and the lowest amount of rainfall was 1 inch so the difference between the highest and lowest amounts of rainfall is 4 inches. The range is the width of the interval that contains all of the data. The average amount of rain in the southern states for this month and year differs by up to four inches.

c. Estimate the numerical value of the "standard deviation," and describe the information it could provide.

Sample Answers: The value of the standard deviation is approximately 1.2 inches. Descriptions about what information the standard deviation provides will vary according to the knowledge about standard deviation students have prior to this activity. For example, some students might not know much while others might conjecture that the standard deviation might be how far the values are from each other, or that it is a common unit of difference from the center.

2. The arrow on the lower graph can be used to generate other possible distributions of rainfall amounts for that month in those 15 states that might, for example, have come from different years. Investigate how the distributions and associated summary measures change.

a. What measure(s) remains constant? Explain why. What would this indicate about the amount of rainfall in those states for the different years?

Sample Answers: Mean and range remain constant since all the plots are symmetric and centered around a value of 3. The highest and lowest rainfall amounts remain the same on each plot as well. Students might also answer that the median remains constant. Keeping the mean and range constant would indicate that over the years, the rainfall did not vary that much overall for the southern states.

Teacher Tip: You might want to discuss that in a real context, the amounts for the states might vary more than is suggested here, but often overall patterns do not change that much.

b. What measure(s) changes? Explain the relationship between the measure(s) and the distribution.

Answer: The standard deviation changes. The distribution with the larger standard deviation has more data farther away from the mean. Students might also answer that the interquartile range changes.

c. Of the distributions you examined, why does the initial distribution have the smallest standard deviation?

Sample Answers: The initial distribution of the amount of rainfall has the smallest standard deviation because more data are closer to the mean.

d. Describe a distribution with the largest possible standard deviation for 15 data values that have a mean of 5, a minimum of 0, and a maximum of 10. Explain your reasoning.

Sample Answers: The distribution will be symmetric and bimodal with seven values of 0, seven values of 10, and one value located at the mean 5. Since the data are symmetric, the mean is 5. Since all of the data, with the exception of the one value at 5, are as far away from the mean as possible, the standard deviation would be the largest possible.

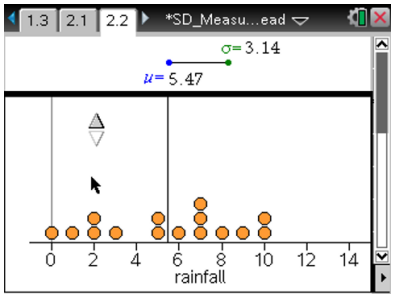
TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

Move to page 2.2.

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☜) getting ready to grab the point. Then press ctrl ☜ to grab the point and close the hand (☜). To de-select a point, move the cursor to a white space on the screen, and click. Be sure students recognize that unless they de-select a point they have moved, it will move along with the next point they choose.

Tech Tip: You might want to demonstrate that clicking on the vertical lines reveals the associated plot values.

3. The dot plot shows a possible distribution of the hypothetical total amount of rainfall for a given month in inches in a given year for the same 15 Southern states. You can move the points to change the possible distributions and investigate how the mean and the standard deviation change. The arrow resets the dot plot.



a. Sketch what you think a distribution with a standard deviation of approximately 0 inches would look like. Then create such a distribution by dragging the points. How well did it match your conjecture?

Sample Answers: The values will probably be close together if the standard deviation is about 0 inches.

b. Describe the distribution, and interpret it in the context of the rainfall.

Sample Answers: All of the data values are the same in the distribution, indicating that every state had the same amount of rainfall.

Teacher Tip: There is no spread around the center, hence a standard deviation of 0. More than one distribution can have a standard deviation of 0; this might be a good opportunity to have a discussion with students about what it takes to have a standard deviation of 0.

4. a. How will the standard deviations compare if 1) distribution A has 14 data points with a value of 10 inches and one data point with a value of 0 inches, and 2) distribution B has 14 data points with a value of 7 inches and one data point at 0 inches. Explain your reasoning. Drag the points to verify your prediction.

Sample Answers: The mean will be very close to 10 inches (~9.33) for distribution A, and the standard deviation will be about 3 inches since all but one data point will be close to the mean. For distribution B, the mean will be smaller, say about 6.5 inches, because the center cluster is lower; the standard deviation will also be smaller, about 1.7, because 0 (the outlier) is not as far away from the mean as in distribution A.

b. In question 2d, you described a distribution with the largest possible standard deviation for 15 possible data values that have a mean of 5, a minimum of 0, and a maximum of 10. Verify your response to 2d by dragging points on the dot plot. Were you surprised? Why or why not?

Sample Answers: Students might or might not be surprised depending on their reasoning. Answers will depend on how accurately students position the points.

c. If you allow the mean to vary between 0 and 10, do you think you can get a standard deviation larger than the one you found in your answer to question 4b? Explain your reasoning, and then test your conjecture by dragging the points.

Sample Answers: Some students might suggest that no matter how they drag the points, they cannot get a standard deviation larger than the one they found in question 2d. Others will notice that they can get a standard deviation of approximately 4.9888 by dragging eight points to 10 and seven to 0. If 8 points were at 0 and 7 points at 10, the standard deviation is 4.9888 as well since the graphs are symmetric to each other. However, the values are really not that different from those in question 4b.

5. Work with a partner to determine whether it is possible to have two different distributions with the same mean and standard deviation. If so, describe a strategy that worked. If not, explain.

Sample Answers: Yes. Starting with a given distribution, moving central points farther away from the mean while moving distant points closer to the mean can create a new distribution with the same mean and standard deviation.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

Teacher Tip: You might want to have students work in pairs for question 5. One student creates a distribution with a certain mean and standard deviation. The students should then work together to create a second distribution that has the same mean and standard deviation.

6. What do you think is the biggest difference between a distribution with a small standard deviation and one with a large standard deviation?

   Sample Answers: The closer the data are to the mean, the smaller the standard deviation will be. The farther the data are away from the mean, the larger the standard deviation will be. Data that create bimodal distributions with peaks far from the mean or distributions with outliers increase the standard deviation.

Wrap Up

Upon completion of the activity and discussion, the teacher should ensure that students are able to understand:

- Standard deviation as a measure of spread of a distribution around the center.
- That the size of the standard deviation depends on the spread of a distribution.
- That maximum or minimum values can affect standard deviation.

Assessment

1. If you have two symmetric distributions with the same mean and range, and one is bimodal and the other is mound-shaped, which would you expect to have the larger standard deviation? Explain.

   Answer: In a mound-shaped symmetric distribution, most of the data are clustered around the mean that is located in the middle of the distribution. A symmetric bimodal distribution could have most of the data at either end of the range and thus, would have a larger standard deviation since more data are farther from the mean.

2. In a given distribution, if one data value is changed to become an outlier, how is the standard deviation affected?

   Answer: The standard deviation increases with outliers.

TI-Nspire Navigator

Note 1

Question 2, *Screen Capture*

Screen captures for question 2 provide the opportunity to discuss data being closer to or farther away from the mean. Students can compare and contrast the different distributions and measures of their means and standard deviations.

Note 2

Question 5, *Live Presenter*

One group can demonstrate on their handheld how to create a different distribution but with the same mean and the same standard deviation as the prior distribution