Binomial Expansion

7 8 9 10 **11** **12**

TI-30XPlus MathPrint™ | Worksheet | 15 min

Calculator Skills:

- Generate a List using the Sequence command
- Use the ${}^{n}C_{r}$ command (Multi-Tap key)

Formula:

$$(ax+by)^n = \sum_{r=0}^{n} {}^{n}C_{r}\,(ax)^{n-r}\,(by)^{r}$$

Don't be spooked by the scary formula, it's really easy to use and each term will be generated separately using lists. Make sure you watch the video first!

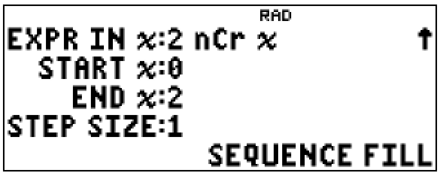
Question: 1.

In this question we expand expressions of the form $(x+y)^n$ (by hand) and then look at how the calculator lists can be used to expedite the process.

i) Write $(x+y)^2$ in expanded form.

> Study the exponents of x and y from left to right, this will help you expand much higher degree expressions later.

ii) Use the calculator lists to generate the coefficients of $(x+y)^2$.
The video tutorial demonstrates how to include the term number utilising List 1 and the coefficients in List 2.
An alternative solution is to use the sequence command shown opposite and use only one list.
Note: The sequence command uses 'x' as the variable which we are using to model the exponent (n).



iii) Write $(x+y)^3$ in expanded form, once again, notice the pattern in the exponents of x and y.

iv) Use the lists on the calculator to automatically generate all the coefficients of $(x+y)^3$.

v) Use the patterns observed above and the calculator lists to write: $(x+y)^6$ in expanded form.

Question: 2.

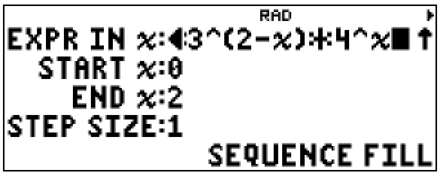
In this question we expand expressions of the form $(ax+by)^n$ (by hand) and then look at how the calculator lists can be used to expedite the process.

i) Write $(ax+by)^2$ and $(3x+4y)^2$ in expanded form.

ii) The general formula to use for each coefficient for expressions of the form: $(ax+by)^n$ is: ${}^{n}C_{r}\times a^{n-r}\times b^{r}$.

For the expression: $(3x+4y)^2$ use: ${}^{2}C_{x}\times 3^{2-x}\times 4^{x}$ (shown opposite using the sequence command.

Compare the list results to the coefficients determined in the previous question. (Q2(i))



iii) Determine the coefficients for $(3x+4y)^4$ by expansion and by editing the sequence formula.

Answers on Page 2

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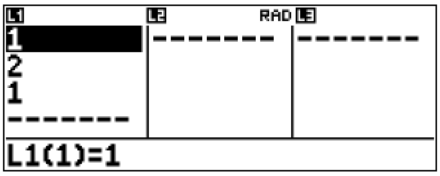
Question: 1.

i) $(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

Coefficients: {1, 2, 1}

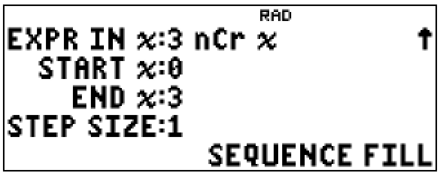


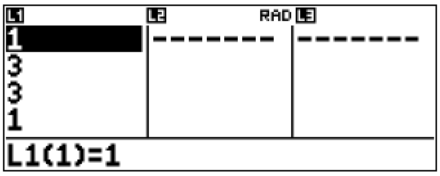
ii) In this example the coefficients of x and y are both 1, this makes the list formula simple. The coefficients are generated using the combinatorics command, row 2 of Pascal's triangle.

iii) $(x+y)^3 = (x^2 + 2xy + y^2)(x+y) = x^3 + 3x^2y + 3xy^2 + y^3$ Coefficients: {1, 3, 3, 1}

Notice the decreasing powers of x : {3, 2, 1, 0} and corresponding increasing powers of y : {0, 1, 2, 3}

iv) Once again, the coefficients of x and y are both 1. The coefficients in the expanded form can be generated using the combinatorics command, row 3 of Pascal's triangle.

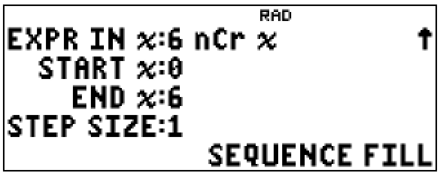


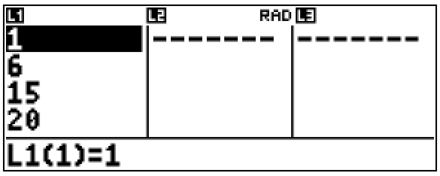


v) $(x+y)^6 = (x^3 + 3x^2y + 3xy^2 + y^3)(x^3 + 3x^2y + 3xy^2 + y^3)$

$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^4 + y^6$

The term by term expansion is exhausting, even using the previous question. The coefficients however are simply the 6th row of Pascal's triangle: {1, 6, 15, 20, 15, 6, 1}, generated using the list command (below). Combining this information with the pattern for the exponents: x: {6, 5, 4, 3, 2, 1, 0} and y: {0, 1, 2, 3, 4, 5, 6}, means it is easy to expand.



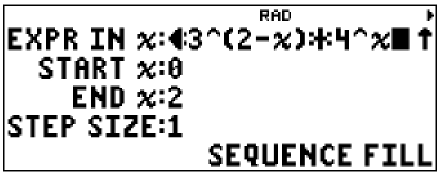


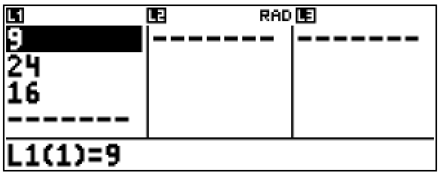
Question: 2.

i) $(ax+by)^2 = a^2x^2 + 2abxy + b^2y^2$ and

$(3x+4y)^2 = 3^2x^2 + 2\times3\times4\times xy + 4^2y^2 = 9x^2 + 24xy + 16y^2$

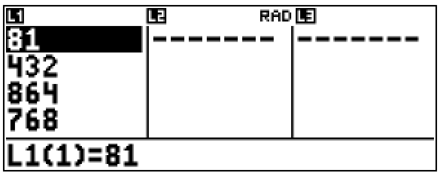
ii) Notice the decreasing powers of the x coefficient are the same as for the powers of x, and similarly with the y coefficient. The x coefficients start at 2 (in general, 'n') and progress to zero, the y coefficients do the reverse.

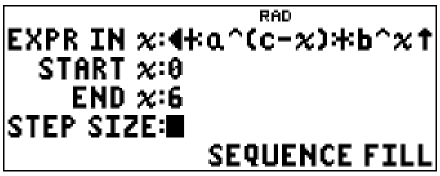




iii) $(3x+4y)^4 = 81x^4 + 432x^3y + 864x^2y^2 + 768xy^3 + 256y^4$. The coefficients generated using the sequence formula in the calculator are: {81, 432, 864, 768, 256}

The formula in the sequence command can be generalised by storing values for 'a' and 'b' and storing the exponent in c. The 'end' value in the sequence still needs to be entered manually.





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