

## Circular/Trigonometric Functions

Each of the questions included here can be solved using either the TI-Nspire CAS.

Scan the QR code or use the link:

### Question 1:

Find the exact solutions to the equation  $4\sin^2(2x) - 3 = 0$  where  $-\pi \leq x \leq \pi$ .

---

---

### Question 2:

Find the first three positive solutions to the equation  $\sqrt{3}\tan(2x) = 1$ .

---

---

### Question 3:

Find the sum of the solutions of  $\sin\left(\frac{x}{2}\right) = \frac{1}{\sqrt{2}}$  for  $0 \leq x \leq 6\pi$ .

---

---

### Question 4:

Find the number of solutions to the equation  $\sqrt{3} + 2\cos\left(\frac{\pi x}{3}\right) = 0$  where  $x \in [-12, 6]$

---

---

### Question 5:

Consider the function  $f(x) = 2\cos\left(\frac{x}{3}\right) + 1$  for  $0 \leq x \leq 6\pi$ . Find the value(s) of  $x$  for which  $f(x) < 0$ .

---

---

## Answers

### Question 1

Solutions:  $x = -\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

The solve( function or zeros( function can be used here. The domain restriction is included at the end of the syntax, as shown below. The advantage of the zeros( function is that our solutions are given in a set, as shown.

Calculator window showing the solve function for the equation  $4(\sin(2x))^2 - 3 = 0$  with domain restriction  $-\pi \leq x \leq \pi$ . The output shows solutions:  $x = -\frac{5\pi}{6}$  or  $x = -\frac{2\pi}{3}$  or  $x = -\frac{\pi}{3}$  or  $x = -\frac{\pi}{6}$  or  $x = \frac{\pi}{6}$  or  $x = \frac{\pi}{3}$  or  $x = \frac{2\pi}{3}$  or  $x = \frac{5\pi}{6}$ .

Calculator window showing the zeros function for the same equation and domain. The output shows the solutions in a set:  $\left\{-\frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}\right\}$ .

### Question 2

Solutions:  $x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}$

You can use the fact that the period of  $y = \tan(2x)$  is  $\frac{\pi}{2}$  and so the first three solutions will be in the domain  $x \in (0, \frac{3\pi}{2})$ . Alternatively you can make use of the general solutions, and find the first three solutions using  $n = 0, 1, 2$ .

Calculator window showing the solve function for the equation  $\sqrt{3} \cdot \tan(2x) = 1$  with domain restriction  $0 < x < \frac{3\pi}{2}$ . The output shows solutions:  $x = \frac{\pi}{12}$  or  $x = \frac{7\pi}{12}$  or  $x = \frac{13\pi}{12}$ .

Calculator window showing the general solution for the same equation:  $x = \frac{(6n+1)\pi}{12}$  for  $n=0, 1, 2$ . The specific solutions for  $n=0, 1, 2$  are also shown:  $x = \frac{\pi}{12}$ ,  $x = \frac{7\pi}{12}$ , and  $x = \frac{13\pi}{12}$ .

### Question 3

Solution:  $12\pi$

You can use the solve function to find the solutions and then add them to find the sum. Alternatively you can use the zeros( function and then use sum(

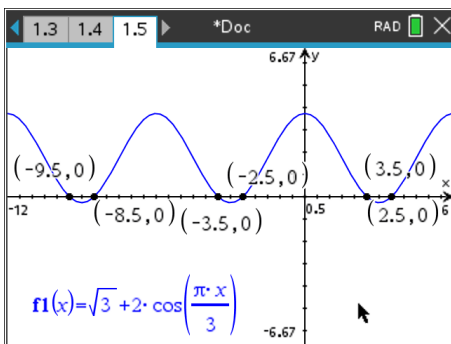
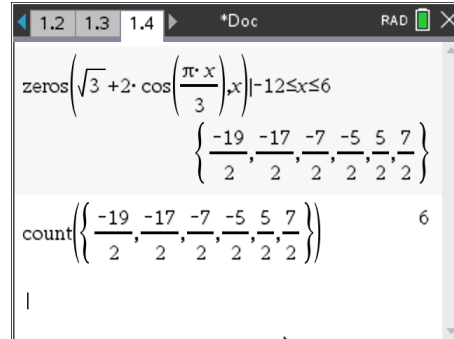
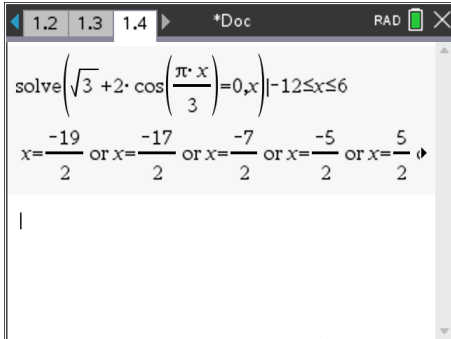
Calculator window showing the solve function for the equation  $\sin\left(\frac{x}{2}\right) = \frac{1}{\sqrt{2}}$  with domain restriction  $0 \leq x \leq 6\pi$ . The output shows solutions:  $x = \frac{\pi}{2}$  or  $x = \frac{3\pi}{2}$  or  $x = \frac{9\pi}{2}$  or  $x = \frac{11\pi}{2}$ . Below, the sum of these solutions is calculated as  $12\pi$ .

Calculator window showing the zeros function for the same equation. The output shows the solutions in a set:  $\left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}\right\}$ . Below, the sum of these solutions is calculated as  $12\pi$ .

**Question 4**

Solution: 6

You can use the solve function or view the graph; however, using the zeros( function and then count( is a nice way to find the number of solutions.



**Question 5**

Solution:  $2\pi < x < 4\pi$  or  $x \in (2\pi, 4\pi)$

As with any inequation, it is a good idea to look at a graph in addition to using the calculator page. If you use the solve function to solve an equation, be sure to write your solution as the appropriate interval, rather than simply transcribing from the CAS. If you attempt to solve an inequation and the CAS gives you general solutions (this happens sometimes!) then remember, you can use the general solution to find specific solutions.

