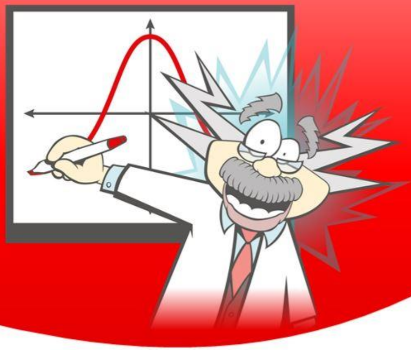
STUDENT REVISION SERIES



Mathematical Methods
Exponential & logarithmic functions

Question 1

The function with rule $f(x) = 2^{-(x+1)} - 3$ is defined over its maximal domain. Write down the domain and range of f.

Question 2

Part of the graph of $y = e^x$ is shown. On the same set of axes, sketch the graphs of

a. $y = e^{-x}$

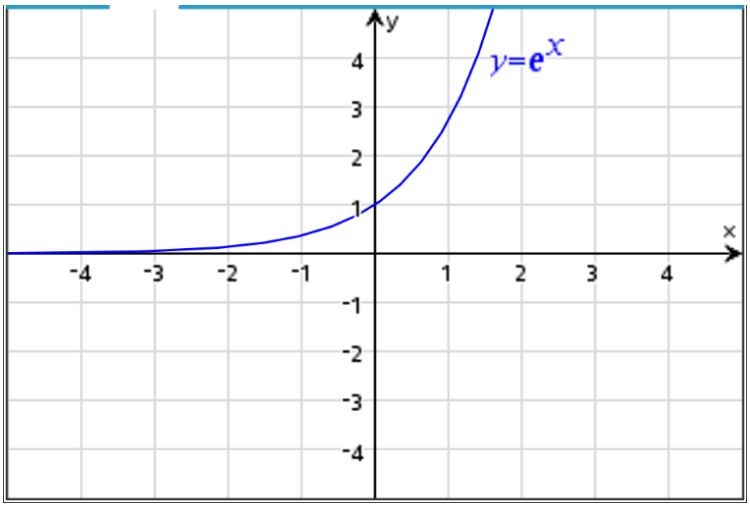
b. $y = -e^x$

c. $y = -e^{-x}$

d. $y = e^x + e^{-x}$

e. $y = -(e^x + e^{-x})$

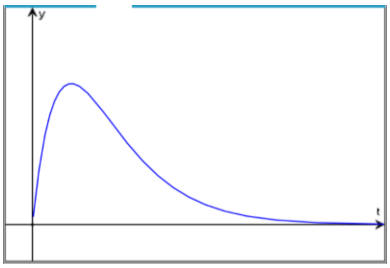
Use TI-Nspire to verify your graphs.



Question 3

Part of the graph of the function $g:[0,\infty) \to R,\ g(t) = 5te^{-\frac{3t}{2}}$ is shown. The range of g is $[0,k]$.

Use a graphical method to find a rational approximation to the value of k, correct to **four decimal places**.



Question 4

Change both sides of the following exponential equations to the same base. Hence determine the value of x.

a. $16^x = \frac{1}{8}$

b. $9^{x-2} = 27$

c. $125^{x-2} = 25^{3x+2}$

Verify your answers using TI-Nspire.

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Question 5

If $y=3^x$, show that $9^x=2\times3^x+3$ can be expressed as $y^2-2y-3=0$.

Solve $y^2-2y-3=0$ for y, and hence find the value(s) of x that satify the equation $9^x=2\times3^x+3$.

Verify your answer using TI-Nspire.

Question 6

Part of the graph of f, where $f(x)=2^x$, is shown. On the same set of axes, use the reflection in the line $y=x$ to sketch the graph of the inverse, $y=f^{-1}(x)$.

Hence also sketch the graphs of

a. $y=f^{-1}(-x)$

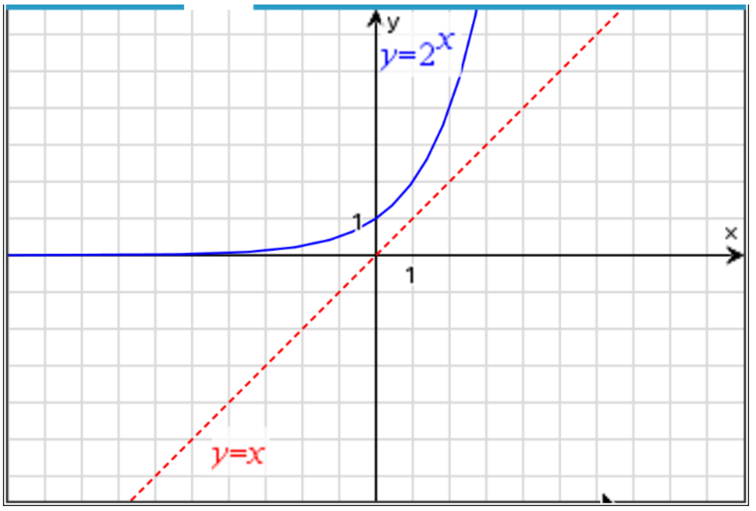
b. $y=-f^{-1}(x)$

The original graph has equation $y=2^x$.

Given $a^y=x \Leftrightarrow y=\log_a(x)$

The inverse graph has equation

$x=2^y \Leftrightarrow y=\log_2(x)$.



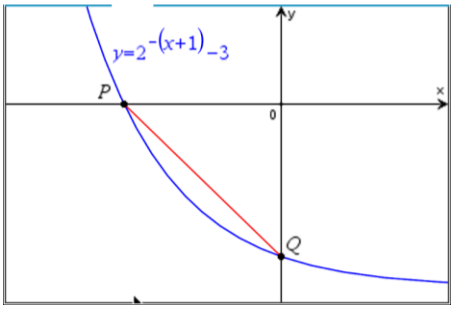
Question 7

Part of the graph of the function f, where $f(x)=2^{-(x+1)}-3$, is shown.

The graph intersects the axes at points P and Q.

a. Determine the coordinates of P and Q.

b. Determine the gradient of the line segment PQ.



Question 8

The electrical charge, C units, stored by an electonic component at time t seconds, is modelled by the function $C(t)=2000\times1.2^t,\ t\geq0$.

a. Determine the initial charge on the component.

b. Determine the time taken for the amount of stored charge to double. Give the answer in seconds, correct to four decimal places.

c. If the rule for C is expressed in the form $C(t)=2000\times e^{kt}$, determine the value of k to four decimal places.

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Logarithm facts

$a^y = x \Leftrightarrow y = \log_a(x)$

$a^{\log_a(x)} = x$

$\log_a(a) = 1$

$\log_a(p \times q) = \log_a(p) + \log_a(q)$

$\log_a\left(\frac{p}{q}\right) = \log_a(p) - \log_a(q)$

$\log_a(x^n) = n\log_a(x)$

Question 9

Use appropriate logarithm facts (see above) to solve the logarithm equations below.

Verify your answers using TI-Nspire.

a. $\log_2(5x-9) = 4$

b. $2\log_e(x) - \log_e(x+3) = \log_e(x-1)$

Question 10 (multiple choice)

If $2\log_e(x) - \log_e(x+3) - \log_e(p) = 0$, then

A. $x = \dfrac{-p \pm \sqrt{p^2+12p}}{2}$

B. $x = \dfrac{p \pm \sqrt{p^2+12p}}{2}$

C. $x = \dfrac{p - \sqrt{p^2+12p}}{2}$

D. $x = \dfrac{p + \sqrt{p^2+12p}}{2}$

E. $x = \dfrac{-p - \sqrt{p^2+12p}}{2}$

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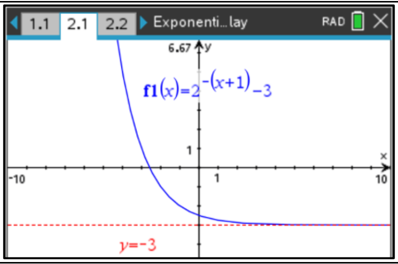
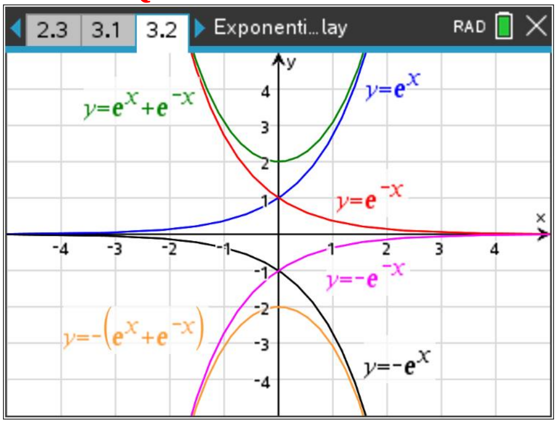
ANSWERS

Answer: Q. 1

Exponential function Domain is R translated 3 units down. Range is $[-3,\infty)$.	

Answer: Q. 2



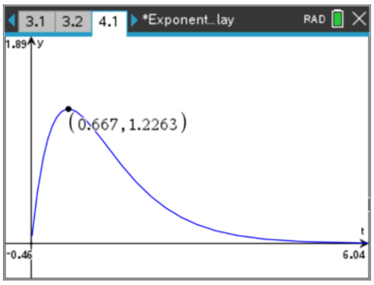
Answer: Q. 3

Range is $[0,k]$

$k=1.2263....$ (correct to 4 decimal places)

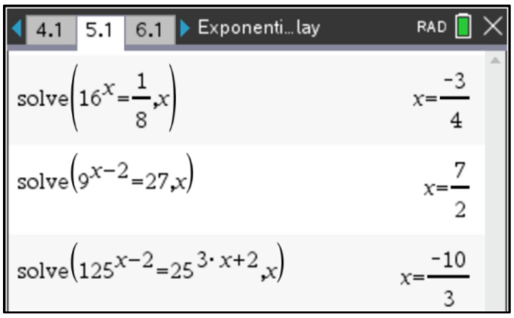
Graphical method. In TI-Nspire 'Graphs' application: Menu > Analyse Graph > Maximum.

Select lower and upper bounds, either side of the highest point on the graph. To increase or decrease decimal places, use the cursor to touch the coordinate and press the '+' or '-' key.



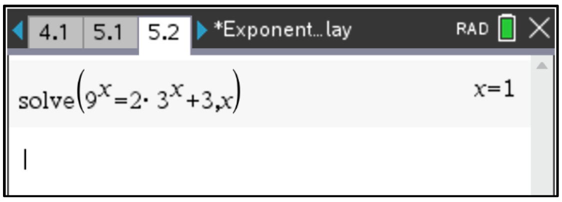
Answer: Q. 4

a.	b.	c.
$16^x=\frac{1}{8}$ $(2^4)^x=2^{-3}$ $4x=-3$ $x=-\frac{3}{4}$	$9^{x-2}=27$ $(3^2)^{x-2}=3^3$ $2x-4=3$ $x=\frac{7}{2}$	$125^{x-2}=25^{3x+2}$ $(5^3)^{x-2}=(5^2)^{3x+2}$ $3x-6=6x+4$ $-10=3x$ $x=-\frac{10}{3}$



Answer: Q. 5

$9^x=2\times3^x+3$ $3^{2x}-2\times3^x-3=0$ $(3^x)^2-2\times3^x-3=0$ Let $y=3^x$ (N.B. $y=3^x>0$ for all x) $y^2-2y-3=0$ (as required)	Solve for y $y^2-2y-3=0$ $(y-3)(y+1)=0$ $y=3$ (reject $y=-1$) $3^x=3$ $x=1$



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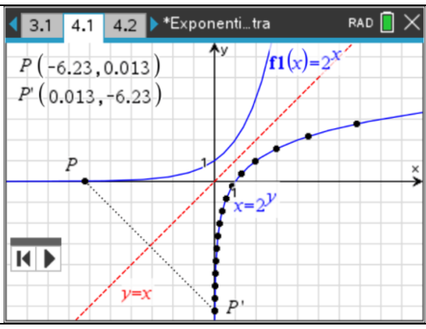
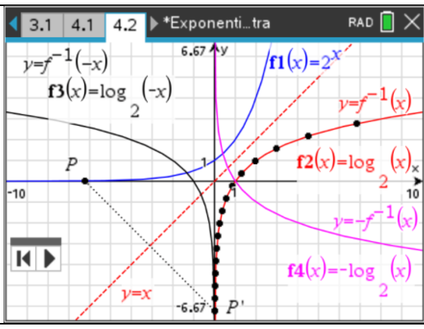
Answer: Q. 6

If $f(x)=2^x$, then the graph of $y=f^{-1}(x)$ is a reflection of $y=2^x$ in the line $y=x$.

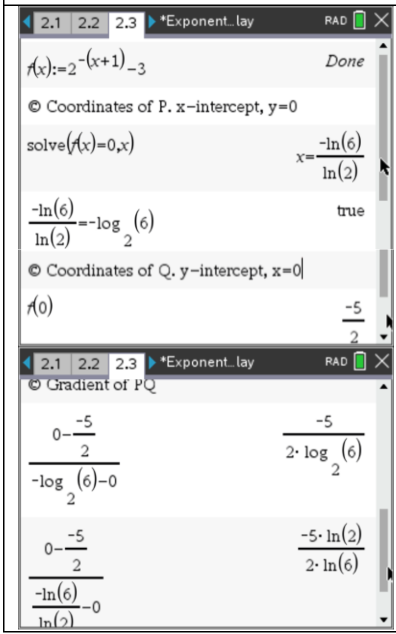
Inverse is given by

$x=2^y \Leftrightarrow y=\log_2(x)$. Therefore, $y=f^{-1}(x)=\log_2(x)$.





Answer: Q. 7

TI-Nspire calculations	Coordinates of P	Coordinates of Q	Gradient of PQ
	x-intercept, $y=0$ **Using TI-Nspire** $P\left(\dfrac{-\log_e(6)}{\log_e(2)},0\right)$ Or equivalent $P(-\log_e(6),0)$ **By-hand** $0=2^{-(x+1)}-3$ $-(x+1)=\log_2(3)$ $x=-1-\log_2(3)$ $x=-(\log_2(2)+\log_2(3))$ $x=-\log_2(6)$ $P(-\log_2(6),0)$	y-intercept, $x=0$ **Using TI-Nspire** $Q\left(0,-\dfrac{5}{2}\right)$ **By-hand** $y=2^{-(0+1)}-3$ $y=2^{-1}-3$ $y=\dfrac{1}{2}-3$ $y=-\dfrac{5}{2}$ $Q\left(0,-\dfrac{5}{2}\right)$	$m=\dfrac{y_p-y_Q}{x_p-x_Q}$ $m=\dfrac{0-\left(-\frac{5}{2}\right)}{-\log_2(6)-0}$ $m=-\dfrac{5}{2\log_2(6)}$ Or equivalent $m=-\dfrac{5\log_e(2)}{2\log_e(6)}$

Answer: Q. 8

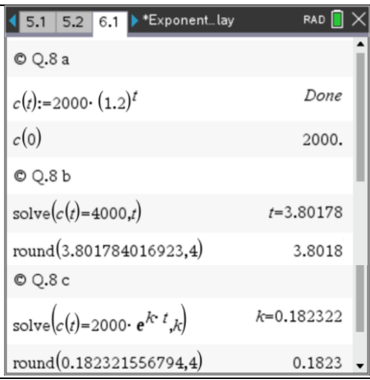
a. $c(0)=2000$

b. Solve $c(t)=4000$ for t

$t=3.8018...$ correct to 4 decimal places

c. Solve $c(t)=2000e^{kt}$ for k

$k=0.1823...$ correct to 4 decimal places



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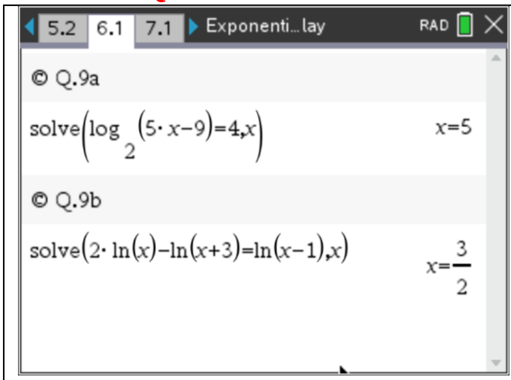
Answer: Q. 9

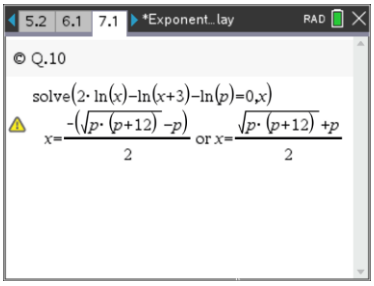
	9a. by-hand $\log_2(5x-9)=4$ $5x-9=2^4$ $5x=16+9$ $5x=25$ $x=5$	9b. by-hand $2\log_e(x)-\log_e(x+3)=\log_e(x-1)$ $\log_e\left(\frac{x^2}{x+3}\right)=\log_e(x-1)$ $\frac{x^2}{x+3}=x-1$ $x^2=(x-1)(x+3)$ $x^2=x^2+2x-3$ $2x=3$ $x=\frac{3}{2}$

Answer: Q. 10

Solving $2\log_e(x)-\log_e(x+3)-\log_e(p)=0$ for x gives solutions $x=\frac{p\pm\sqrt{p(p+12)}}{2}$. However, the solutions comes with a caution symbol!

$p>0$ and $x>0$ because $p,\ x$ are arguments of logarithms.

Therefore, **D.** $x=\frac{p+\sqrt{p^2+12p}}{2}>0$. Reject $x=\frac{p-\sqrt{p^2+12p}}{2}<0$



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