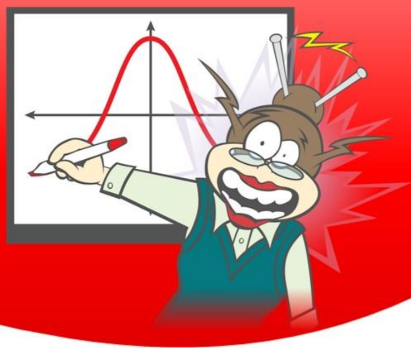
STUDENT REVISION SERIES



Graph Sketching Part 2

Each of the questions included here can be solved using the TI-Nspire CX CAS.

Scan the QR code or use the link: http://bit.ly/GraphSketchingP2

Question: 1.

The graph of $f(x)=\dfrac{1}{x^3-6x^2+3x+10}$ has four asymptotes. Find the equations of these asymptotes.

Question: 2.

If the graph of $g(x)=\dfrac{3}{x^3-4mx^2+2nx}$ has exactly two vertical asymptotes, state n in terms of m.

Question: 3.

Find the coordinates of the stationary points of the graph of $h(x)=\dfrac{16}{x^3-6x^2+9x+16}$ and state their nature.

Author: S Crouch

Question: 4.

Let $f : D \to \mathbb{R},\ f(x) = \dfrac{8}{x^2 - 8x + 12}$.

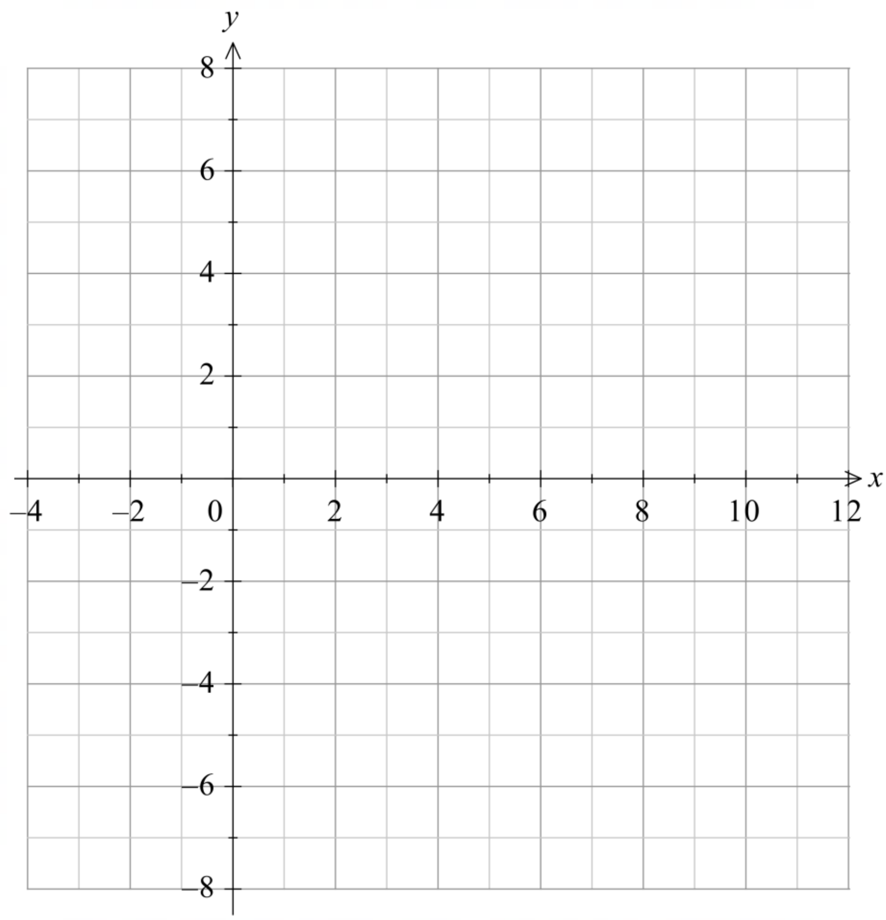
a. State D, the maximal domain of f.

b. State the equations of the asymptotes of the graph of $y = f(x)$.

c. Find the coordinates of all axes intercept(s) of the graph of $y = f(x)$.

d. Use calculus to find the coordinates of all stationary point(s) on the graph of $y = f(x)$, and state their nature.

e. Sketch the graph of $y = f(x)$ on the axes below, labelling axes intercepts and stationary points with their coordinates, and asymptotes with their equations.



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Question: 5.

Express the function $\frac{2x^3-7x^2+2x+5}{x^2-5x+6}$ in the form $ax+b+\frac{a}{x-b}+\frac{b}{x-a}$ and hence state the equations of all asymptotes of the graph of $y=\frac{2x^3-7x^2+2x+5}{x^2-5x+6}$.

Question: 6.

Show that the curve with equation $y=\frac{4x}{x^2-6x+11}$ has no vertical asymptotes.

Question: 7.

Find the coordinates of all stationary point(s) and point(s) of inflection on the graph of $f(x)=\frac{x^3-16}{4x}$.

Question: 8.

Consider the function with rule $y=\frac{8-x^4}{2x^2}$ over the range $y\in\left[-\sqrt{17},\sqrt{17}\right]$. The domain may be expressed in the form $x\in\left[-\sqrt{a+\sqrt{17}},-\sqrt{a-\sqrt{17}}\right]\cup\left[\sqrt{a-\sqrt{17}},\sqrt{a+\sqrt{17}}\right]$, where $a>0$. Find the exact value of a.

Author: S Crouch

Question: 9.

Let $f : D \to \mathbb{R}$, $f(x) = \dfrac{x^3 + 4x^2 - 3x - 18}{x^2 - 4}$.

a. Factorise the numerator and denominator of f, and hence state the maximal domain D.

b. State the equations of the asymptotes of the graph of $y = f(x)$.

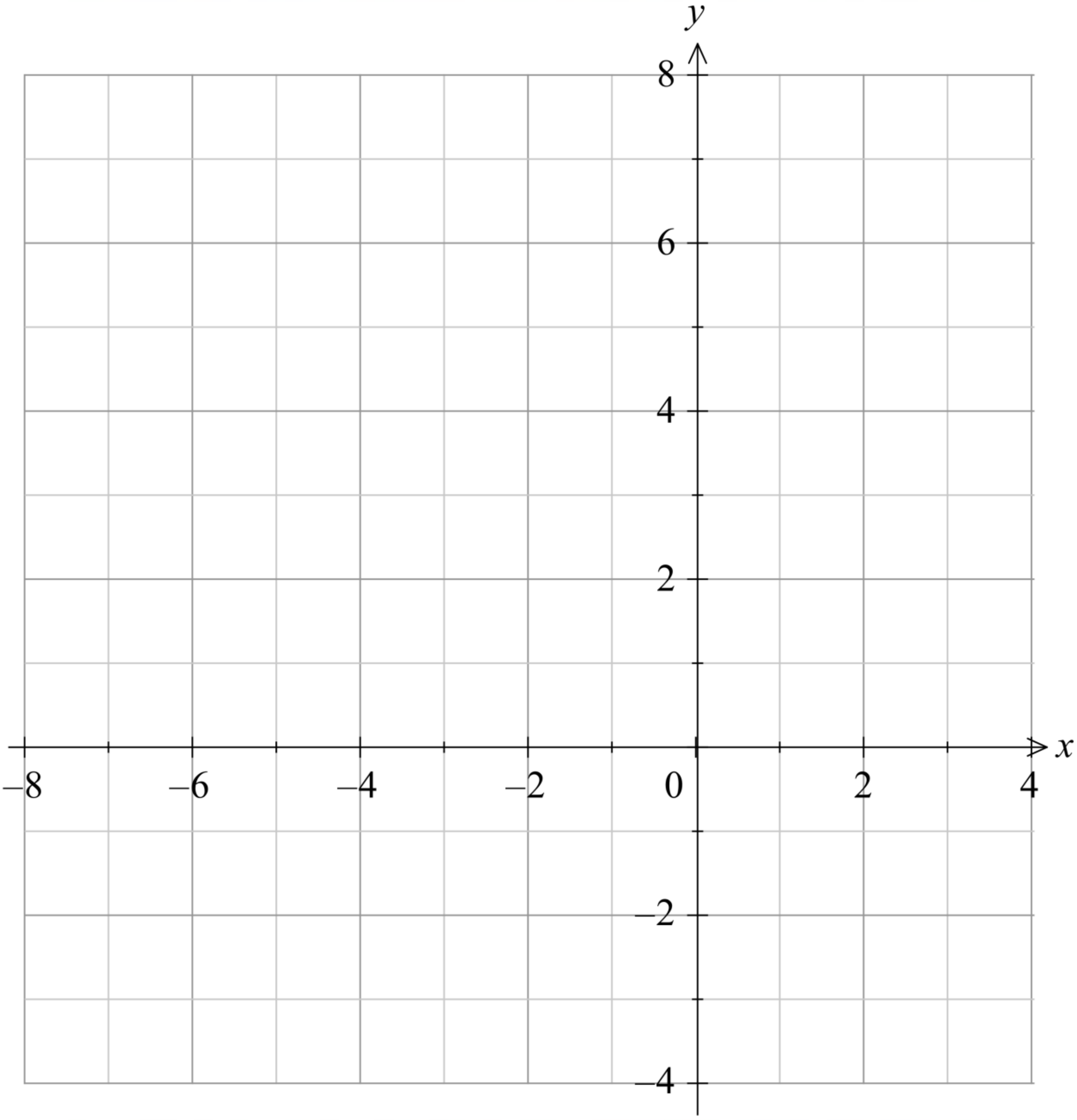
c. Find the coordinates of all axes intercept(s) of the graph of $y = f(x)$.

d. Use calculus to find the coordinates of all stationary point(s) on the graph of $y = f(x)$, and state their nature.

e. Use calculus to show that the graph of f has no points of inflection.

Author: S Crouch

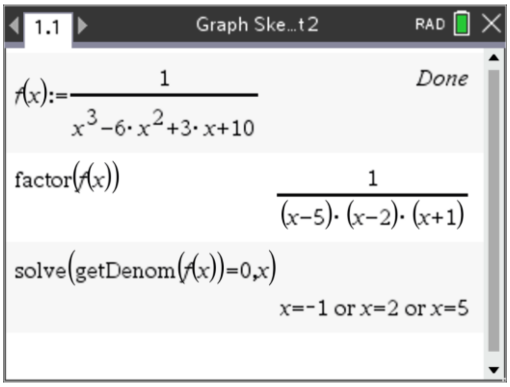
f. Sketch the graph of $y = f(x)$ on the axes below, labelling axes intercepts and stationary points with their coordinates, and asymptotes with their equations. Ensure all discontinuities are shown and labelled.



g. Show that the tangent to the graph of $y = f(x)$ at the point where $x = -\frac{3}{2}$ passes through the origin.

Answers

Question 1 $x=-1,\ x=2,\ x=5,\ y=0$



Define the function. [ctrl] [:=] can be used to get :=

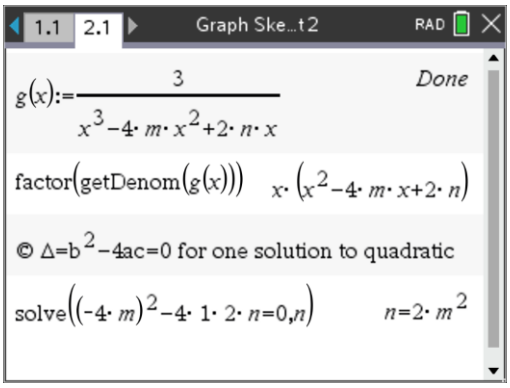
The denominator can be factorised using the factor command ([menu] [3] [2]).

To extract the denominator, use the getDenom command ([menu] [2] [7] [3]).

To find the roots of the denominator, use the solve command ([menu] [3] [1]).

The (vertical) asymptotes of the graph of f are $x=-1$, $x=2$, $x=5$ and the (horizontal) asymptote is $y=5$.

Question 2 $n=2m^2$

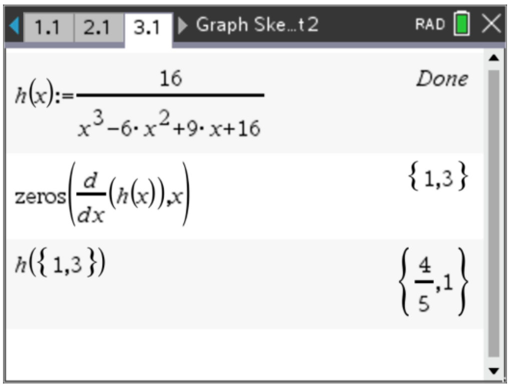


When defining the function, ensure that multiplication signs are placed especially between two variables (such as $m\cdot x^2$ and $n\cdot x$).

When the denominator (a cubic) is factorised, the product of a linear (x) and quadratic ($x^2-4mx+2n$) is obtained.

$x=0$ is therefore always a vertical asymptote, so for a total of two vertical asymptotes, the quadratic is required to have one root (that is, the discriminant is equal to zero).

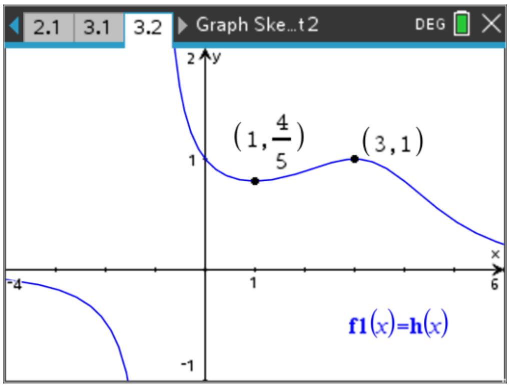
Question 3 Local minimum at $\left(1,\frac{4}{5}\right)$; Local maximum at $(3,1)$



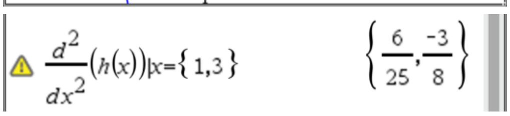
The zeros command ([menu] [3] [4]) provides the roots of an expression as a list.

A quick way to access the derivative command is [shift] [−].

The list outputted from the zeros command can be used as an input into the function to generate all y-values of stationary points at once.



To obtain the nature of the stationary points, a graph of $y=h(x)$ is useful.



Alternatively, the second derivative can be used to find the nature of the stationary points. Recall that:

$h''(a)>0 \Rightarrow (a,h(a))$ is a local minimum

$h''(a)<0 \Rightarrow (a,h(a))$ is a local maximum

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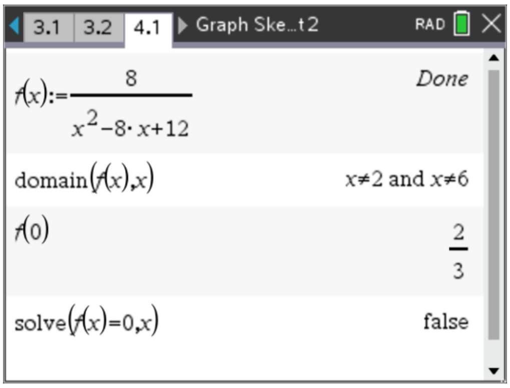
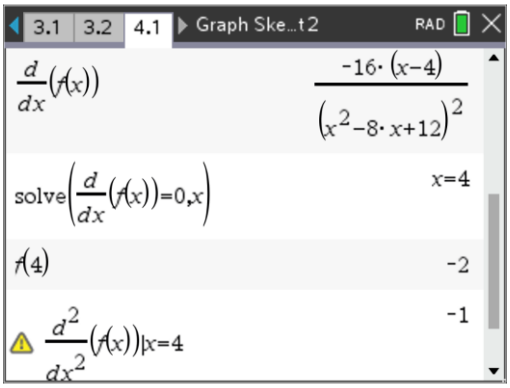
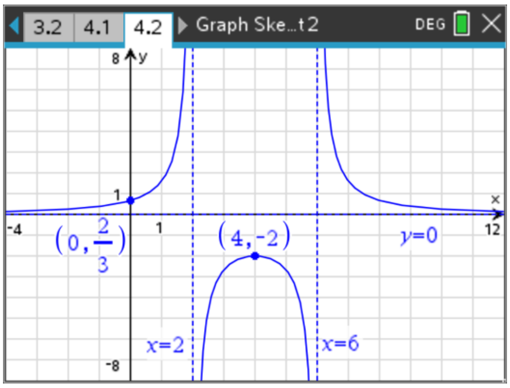
Question 4







a. $D=\mathbb{R}\setminus\{2,6\}$

The maximal domain of f is $\text{dom } f=\mathbb{R}\setminus\{2,6\}$.

The domain command can be obtained from the catalog ( ).

b. The horizontal asymptotes are $x=2$ and $x=6$. The vertical asymptote is $y=0$.

c. $\left(0,\dfrac{2}{3}\right)$

There is only one axis intercept: the y-intercept. The graph of $y=f(x)$ does not have any x-intercepts.

d. Local maximum at $(4,-2)$

$f'(x)=0$ when $x=4$, and $f(4)=-2$.

$f''(4)=-1<0$ so local maximum at $(4,-2)$.

e. See CAS graph.

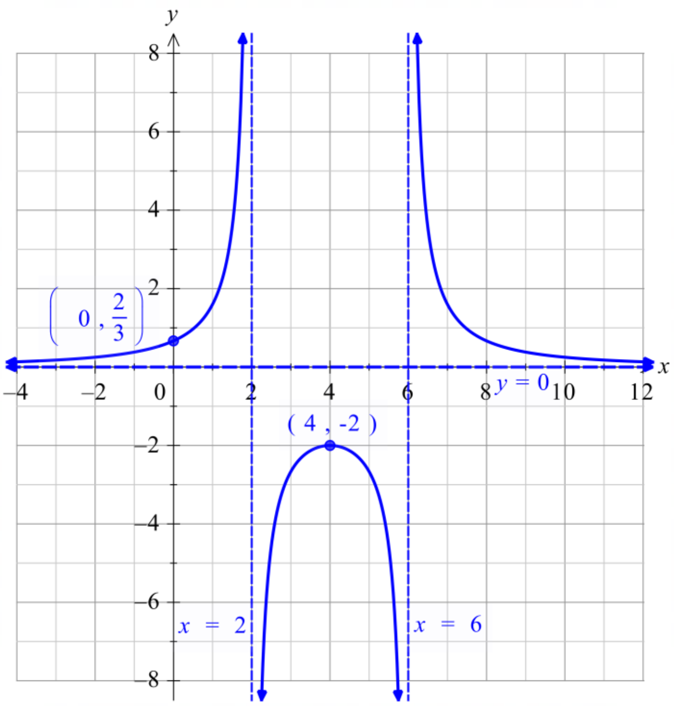
Relation mode (menu 3 2) is used for graphing asymptotes $x=2$ & $x=6$.

Trace (menu 5 1) is used for plotting the y-intercept.

To plot the stationary point, use Maximum (menu 6 3).

After the Window Settings are changed (to match the axes provided in the question), grid lines can be shown (menu 2 6 3).

This ensures that the CAS screen resembles the provided axes and desired output as closely as possible.



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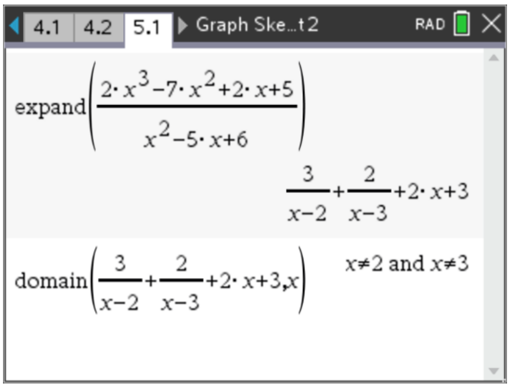
Question 5 $x=2,\ x=3,\ y=2x+3$



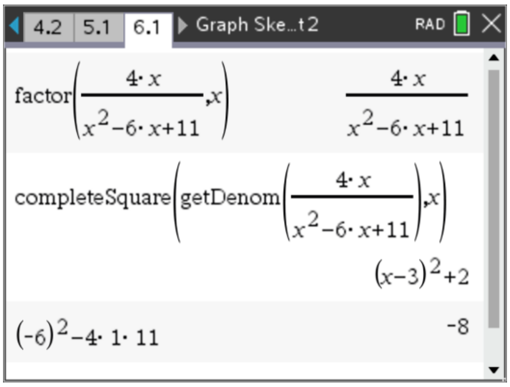
The expand command (menu 3 3) is used to simplify the fraction by performing both division and partial fraction decomposition.

As $x \to \pm\infty$, both $\dfrac{3}{x-2}$ and $\dfrac{2}{x-3}$ approach 0, hence the oblique asymptote is $y=2x+3$.

The domain command can be used to find the equations of vertical asymptotes, which for this function are $x=2$ and $x=3$.

A graph may be used to verify these asymptotes.

Question 6



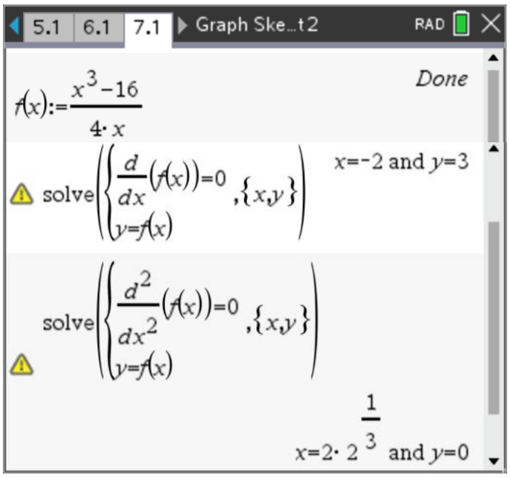
There are multiple ways to show that the given curve has no asymptotes.

The factor command can be used, which shows that the expression is not factorisable.

The denominator can also be extracted and converted to turning point form, showing that the quadratic has no real roots.

Finally, the discriminant of the denominator can be evaluated (negative).

Question 7 Stationary point $(-2,3)$ [Local minimum] and Point of inflection $\left(2\sqrt[3]{2},0\right)$

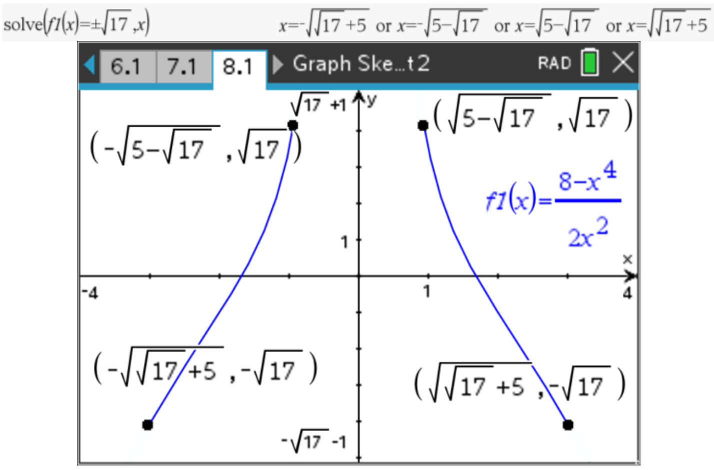


Define $f(x)$ in the usual way.

To find the coordinates of stationary points, solve a set of simultaneous equations with $f'(x)=0$ and $y=f(x)$. This will output the stationary points as ordered sets of x & y values which are coordinates.

To find the coordinates of points of inflection, solve a set of simultaneous equations with $f''(x)=0$ and $y=f(x)$. This will output the stationary points as ordered sets of x & y values which are coordinates.

Question 8 $a=5$



Solving $f(x)=0$ for x results in $x=\pm\sqrt{5-\sqrt{17}}$ and $x=\pm\sqrt{5+\sqrt{17}}$.

Sketching the graph of $y=f(x)$ over the domain $x\in\left[-\sqrt{5+\sqrt{17}},-\sqrt{5-\sqrt{17}}\right]\cup\left[\sqrt{5-\sqrt{17}},\sqrt{5+\sqrt{17}}\right]$ shows that the range of the graph is $y\in\left[-\sqrt{17},\sqrt{17}\right]$ as expected.

Therefore $a=5$.

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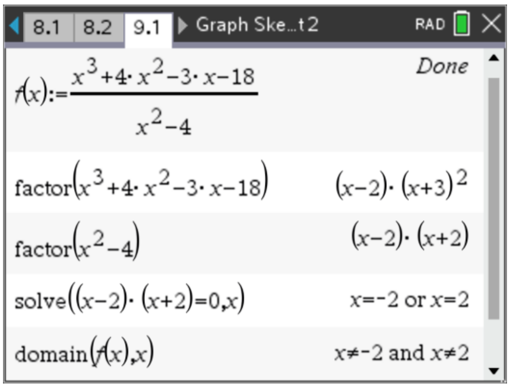
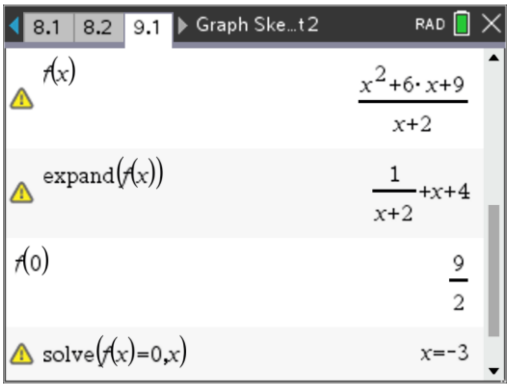
Question 9



a. $(x-2)(x+3)^2$; $(x-2)(x+2)$; $D=\mathbb{R}\setminus\{-2,2\}$

The numerator and denominator share a common factor $(x-2)$, therefor they cancel out in $f(x)$, causing a point of discontinuity (a "hole") in the graph of $y=f(x)$.

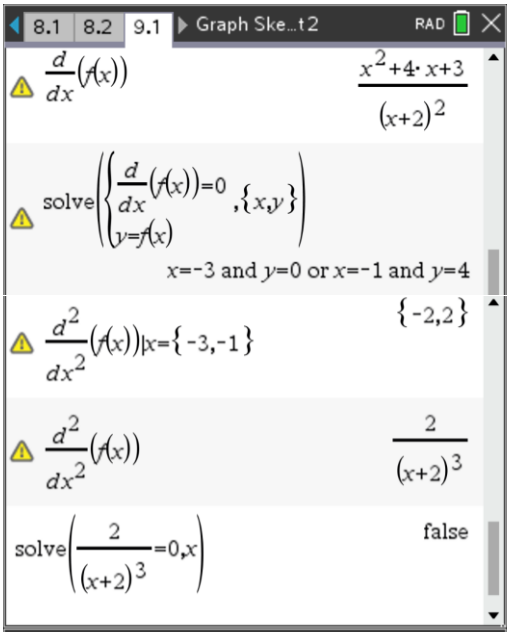
Furthermore, due to the presence of $(x+2)$ in the denominator, this leac to a vertical asymptote with equation $x=-2$. Hence, the implied domain D is $D=\mathbb{R}\setminus\{-2,2\}$.



b. $x=-2$, $y=x+2$

$f(x)$ simplifies to $f(x)=\dfrac{x^2+6x+9}{x+2}$. This can be expanded via lor division or the expand command to give the equations of the asymptotes a: $y=x+4$ (oblique) and $x=-2$ (vertical).

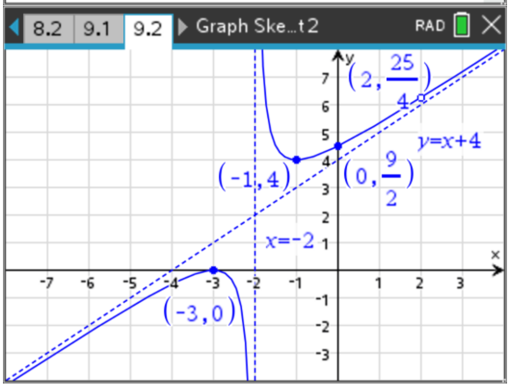
c. The y-intercept is $\left(0,\dfrac{9}{2}\right)$ and the x-intercept is $(-3,0)$



d. Local maximum at $(-3,0)$, Local minimum at $(-1,4)$

Using calculus, $f'(x)=\dfrac{x^2+4x+3}{(x+2)^2}$. The roots of the first derivative ar $x=-3$ and $x=-1$. The second derivative evaluated at these x-values respectively are negative and positive respectively.

e. Using calculus, $f''(x)=\dfrac{2}{(x+2)^3}$, which has no solutions when equate to 0. Therefore there are no points of inflection on the graph of $y=f(x)$.



f. See CAS graph.

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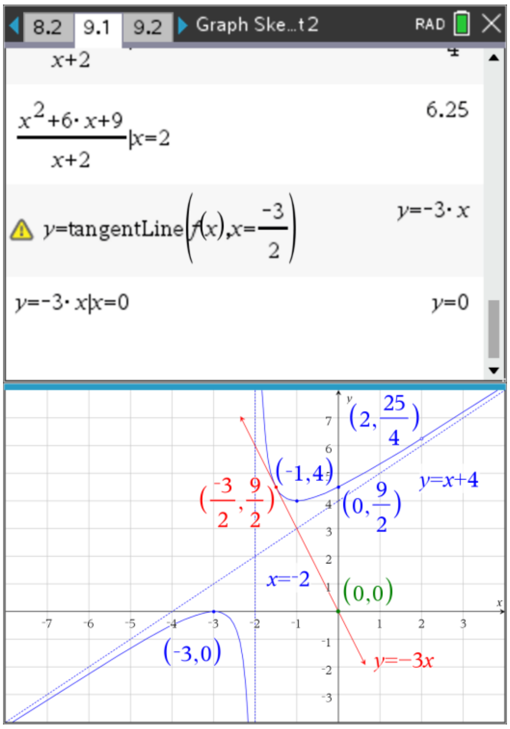
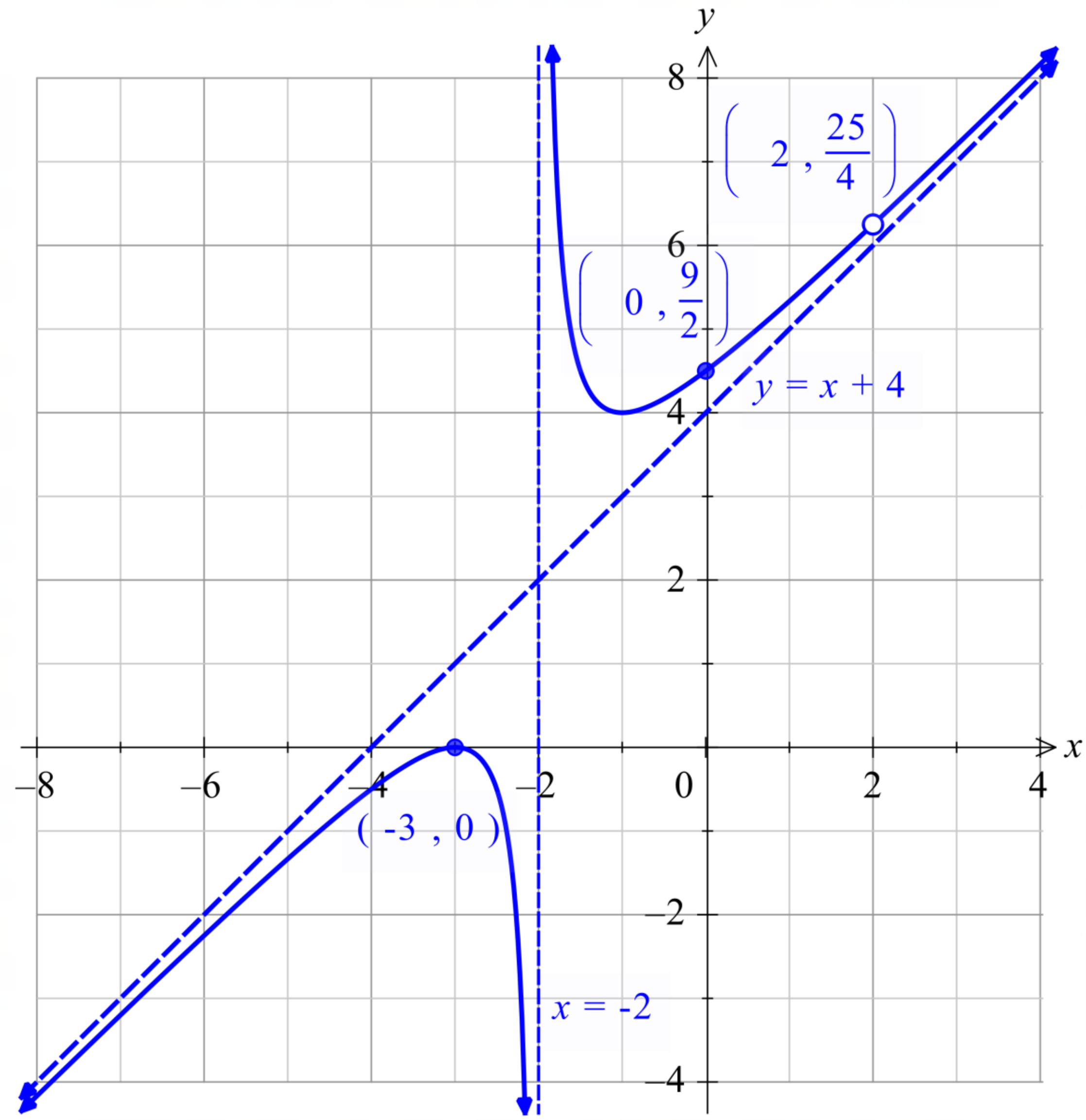


g. $y=-3x$; $x=0 \Rightarrow y=0$ so tangent passes through origin $O(0,0)$

The tangentLine command (menu 4 9) can be used to find the equation of the tangent to the graph of $y=f(x)$ at the point where $x=-\frac{3}{2}$, ensuring that the command is preceded by " $y=$". Then, $x=0$ may be substituted in to confirm that the tangent does indeed pass through $O(0,0)$.

The tangent can also be added to the graph of $y=f(x)$, by pressing menu 8 1 8 and placing the tangent on the point where $x=-\frac{3}{2}$. It may be necessary to edit the x-value to exactly $-\frac{3}{2}$ if required.

Following this, a point can be placed on the origin showing the result.



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