STUDENT REVISION SERIES

Transformations & Composite Functions

Question 1

Describe a sequence of transformations that maps the graph of $y=\sqrt{x}$ onto the graph of $y=-\sqrt{x+2}-1$.

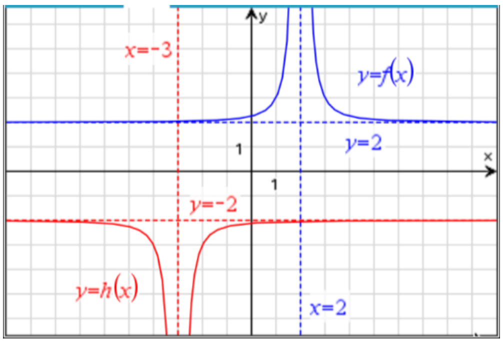
Question 2

The transformation $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} c \\ d \end{bmatrix}$ maps the graph of $y=\sqrt{x}$ onto the graph of $y=-\sqrt{x+2}-1$. Determine the values of a,b,c and d. Verify using the Transformation Matrix Template provided.

Question 3

Parts of the graphs of functions f and h are shown on the right. $T: R^2 \to R^2$ transforms the graph of f onto the graph of h by reflection and translation. If the rule of T is of the form

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} -1 & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} c \\ d \end{bmatrix}$, find possible values of b, c, d.



Question 4

Consider again the graphs of functions f and h shown in Question 3 above. If the rule of T is written in the form

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix}+\begin{bmatrix} p \\ q \end{bmatrix}\right)$, find possible values of p, q.

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Question 5 (multiple choice)

The transformation $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ maps the line with equation $x - 2y - 4 = 0$ onto the line with equation

A. $x - y - 5 = 0$
B. $x + y + 5 = 0$
C. $x + y + 9 = 0$
D. $x - 2y - 9 = 0$
E. $x + 2y + 5 = 0$

Question 6 (multiple choice)

The transformation $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$ maps the graph of $y = -(2x+1)^2 - 3$ onto the graph of $y = x^2$. Possible values of a, b, c, d are

A. $a = 2, b = -1, c = 1, d = -3$
B. $a = 2, b = -1, c = -1, d = 3$
C. $a = \frac{1}{2}, b = -1, c = 1, d = -3$
D. $a = \frac{1}{2}, b = -1, c = -1, d = 3$
E. $a = \frac{1}{2}, b = -1, c = -1, d = -3$

Question 7 (multiple choice)

Consider the function $f: [0,6] \to R,\ f(x) = 4x + 1$. The transformation $T: R^2 \to R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ maps the graph of f onto the graph of g. The rule and domain of g, respectively, are

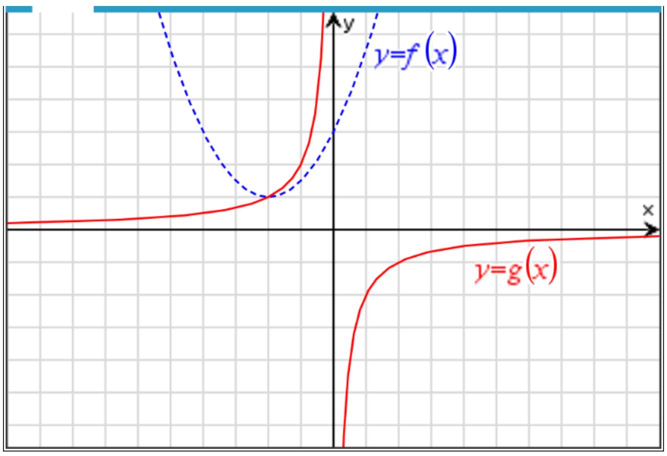
A. $g(x) = -2x + 4,\ x \in [0,6]$
B. $g(x) = -2x - 2,\ x \in [0,6]$
C. $g(x) = -2x + 4,\ x \in [-12,0]$
D. $g(x) = -8x - 2,\ x \in [-12,0]$
E. $g(x) = -8x - 2,\ x \in [0,6]$

Question 8

The functions f and g, with rlues $f(x) = \sqrt{x+2}$ and $g(x) = x^2 + 1$, are defined over their maximal domains. If $h(x) = g(f(x))$, find the rule and domain of h.

Question 9

Parts of the graphs of $y = f(x)$ and $y = g(x)$ are shown below. On the same set of axes, sketch the graph of $y = g(f(x))$.



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Question 10

If the function f satisfies the functional equation $f\left(f\left(x\right)\right)=x$ over its maximal domain, then the rule of f could be

A. $f\left(x\right)=x-1$

B. $f\left(x\right)=\sqrt{x^2+1}-1$

C. $f\left(x\right)=\frac{-2}{x}$

D. $f\left(x\right)=\frac{x-1}{x+1}$

E. $f\left(x\right)=\frac{x^2-1}{x-1}$

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ANSWERS

Answer: Q. 1

Reflect in the x-axis $\left(\sqrt{x} \to -\sqrt{x}\right)$, translate 2 units left $\left(-\sqrt{x} \to -\sqrt{x-(-2)}\right)$, translate 1 unit down $\left(-\sqrt{x+2} \to \left(-\sqrt{x+2}\right)-1\right)$

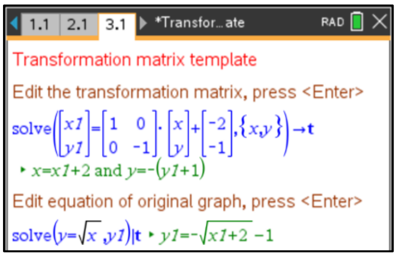
Answer: Q. 2

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a=1 & 0 \\ 0 & b=-1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c=-2 \\ d=-1 \end{bmatrix}$$

$b=-1$: reflection in the x-axis i.e. $(y'=-y)$

$c=-2$: translate 2 units left i.e. $(x'=x-2)$

$d=-1$: translate 1 units down i.e. $(y'=-y-1)$



Answer: Q. 3

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a=-1 & 0 \\ 0 & b=-1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c=-1 \\ d=0 \end{bmatrix}$$

$a=-1$ (given): $(x'=-x)$ reflection in the y-axis

$b=-1$: $(y'=-y)$ reflection in the x-axis

$c=-1$: $(x'=-x-1)$ translation 1 unit left, after the reflection in the y-axis

$d=0$: $(y'=-y+0)$ zero vertical translation

Answer: Q. 4

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} p=1 \\ q=0 \end{bmatrix}\right)$$

$p=1$: $(x'=x+1)$ translation 1 unit RIGHT, before the reflection in the y-axis

$q=0$ (and zero vertical translation)

Then reflect in y-axis and in x-axis.

Answer: Q. 5

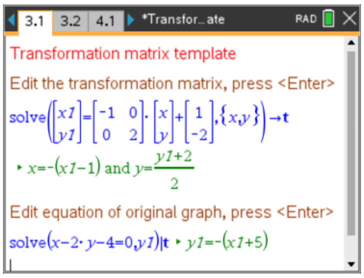
$x'=-x+1 \Rightarrow x=-(x'-1)$

$y'=2y-2 \Rightarrow y=\dfrac{y'+2}{2}$

Substitute into original equation.
Equation of image is

$y=-(x+5)$

B. $x+y+5=0$



Answer: Q. 6

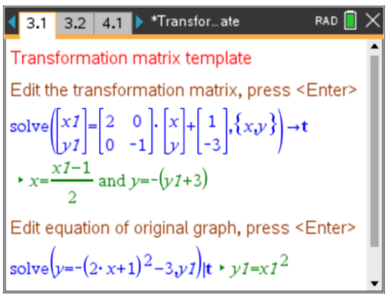
$\left(-(2x+1)^2-3\right) \to \left((x')^2\right)$ requires that $a=2$ and $c=1$, so that in the first step,

$x'=2x+1 \Rightarrow x=\dfrac{x'-1}{2}$

Therefore answer

A. $a=2, b=-1, c=1, d=-3$

Verified using the transformation matrix template.



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Answer: Q. 7

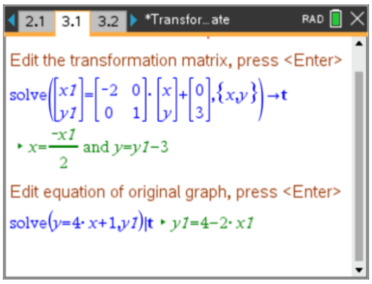
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

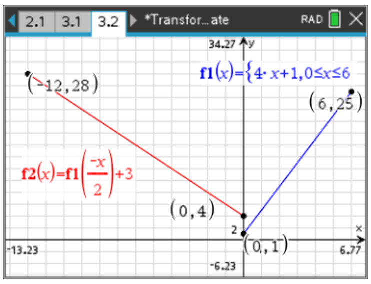
$$x' = -2x,\ y' = y + 3 \Leftrightarrow x = \frac{x'}{-2},\ y = y' - 3.$$

$$y' - 3 = f\left(\frac{x'}{-2}\right)$$

$$y' = f\left(\frac{x'}{-2}\right) + 3 = \left(4\left(\frac{x'}{-2}\right) + 1\right) + 3$$

C. $g(x) = -2x + 4,\ x \in [-12, 0]$





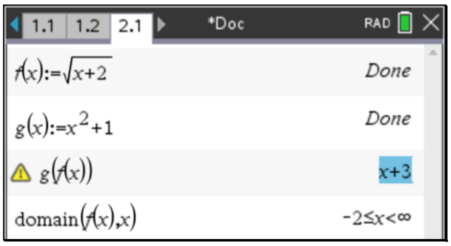
Domain. Since $x' = -2x$, $[0,6] \to [-12,0]$

Answer: Q. 8

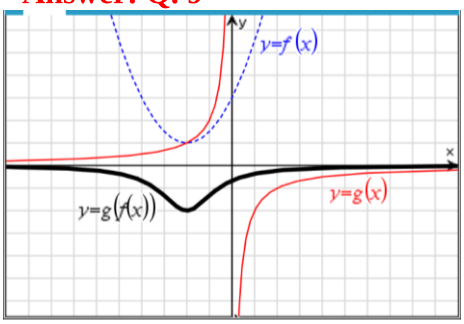
$$g(f(x)) = \left(\sqrt{x+2}\right)^2 + 1 = x + 3$$

Rule: $h(x) = x + 3$

Domain: $d_h = d_f = [-2, \infty)$



Answer: Q. 9



Use TI-Nspire to verify.

$f(x)$ could be $a(x-2)^2 + 1,\ 0 < a < 1$.

Guess a value for a. Input as graph $f1(x)$.

$g(x)$ could be $-\frac{p}{x},\ p > 1$.

Guess a value for p. Input as graph $f2(x)$.

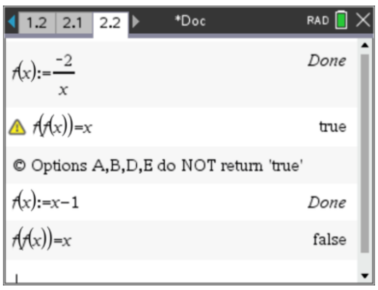
Graph $f3(x) = f2(f1(x))$.

Answer: Q. 10

If $f(x) = \frac{-2}{x}$, then

$$f(f(x)) = \frac{-2}{\left(\frac{-2}{x}\right)} = \frac{-2}{1} \times \frac{x}{-2} = x$$

C. $f(x) = \frac{-2}{x}$ Verify using TI-Nspire



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