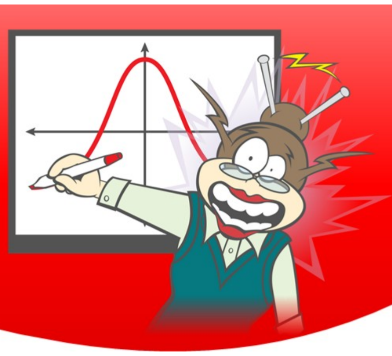

Reciprocal and Inverse Trigonometric Functions

Question: 1.

The maximal domain of the function $f(x) = 3\cos^{-1}\left(\frac{2x-1}{4}\right) + \pi$ is:

A. $\left[-\frac{5}{2}, \frac{3}{2}\right]$ B. $\left[-\frac{3}{2}, \frac{5}{2}\right]$ C. $[\pi, 4\pi]$ D. $\left[-\frac{5}{2}, \frac{11}{2}\right]$ E. $[-1, 1]$

Question: 2.

The maximal domain and range of the function $f(x) = 4\sin^{-1}(3x+1) + \frac{\pi}{2}$ are respectively:

A. $\left[\frac{-3\pi}{2}, \frac{5\pi}{2}\right]$ and $\left[0, \frac{2}{3}\right]$

B. $\left[\frac{-3\pi}{2}, \frac{5\pi}{2}\right]$ and $\left[-\frac{2}{3}, 0\right]$

C. $\left[-\frac{2}{3}, 0\right]$ and $\left[\frac{\pi-8}{2}, \frac{\pi+8}{2}\right]$

D. $\left[0, \frac{2}{3}\right]$ and $\left[\frac{-3\pi}{2}, \frac{5\pi}{2}\right]$

E. $\left[-\frac{2}{3}, 0\right]$ and $\left[\frac{-3\pi}{2}, \frac{5\pi}{2}\right]$

Question: 3.

Find the sum of the solutions to $\sin^2(2x) = \frac{1}{4}$ given $0 \le x \le \pi$.

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Question: 4.

The asymptotes of $y = \frac{1}{3}\tan^{-1}(3x)$ are given by:

A. $y = \pm\frac{\pi}{2}$ B. $x = \pm\frac{\pi}{2}$ C. $y = \pm\frac{\pi}{6}$ D. $x = \pm\frac{\pi}{6}$ E. $y = \pm\frac{3\pi}{2}$

Question: 5.

Consider the function $f(x) = a\sin^{-1}(x + b)$. Given that f has domain $[-3, -1]$ and range $[-\pi, \pi]$, it follows that:

A. $a = 2,\ b = -2$

B. $a = 2,\ b = 2$

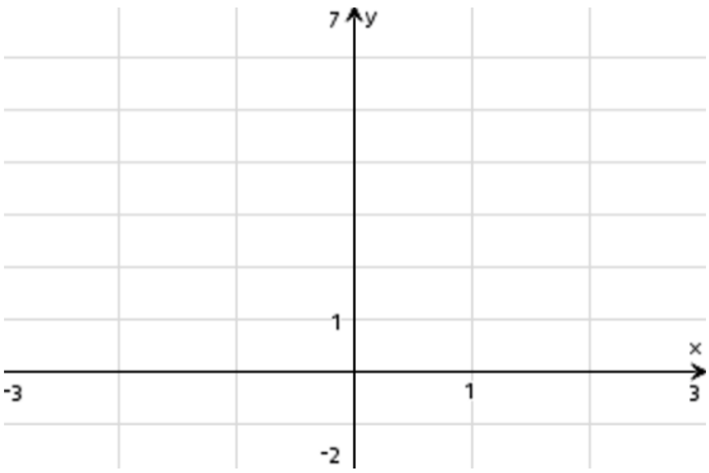
C. $a = -2,\ b = 2$

D. $a = -2,\ b = -2$

E. $a = \pi,\ b = -\pi$

Question: 6.

Consider the function with rule $f(x) = 3\arccos\left(\sqrt{1 - x}\right)$. Sketch the graph of f over its maximal domain. Label the endpoints with their coordinates.



Question: 7.

Find the coordinates of the points of intersection of the graph with the equation $y = \operatorname{cosec}^2\left(\frac{\pi x}{8}\right)$ and the line $y = \frac{4}{3}$ for $0 < x < 8$.

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Question: 8.
The implied domain of the function with rule $f(x) = 1 + b\arcsin(ax)$ where $a > 0$ is:

A. $(-\infty,\ \infty)$

B. $\left[\frac{1}{a}, -\frac{1}{a}\right]$

C. $\left[-\frac{1}{a}, \frac{1}{a}\right]$

D. $[1-b,\ 1+b]$

E. $[-1,\ 1]$

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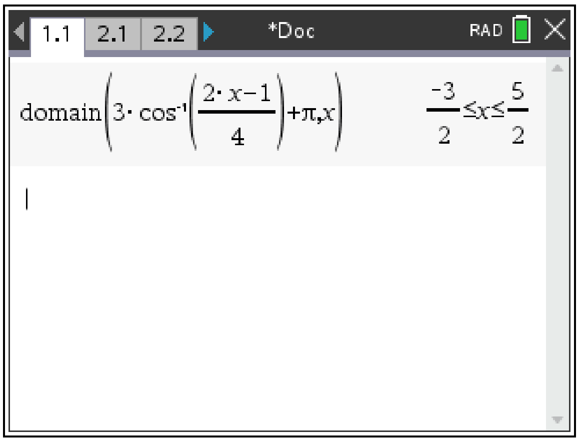
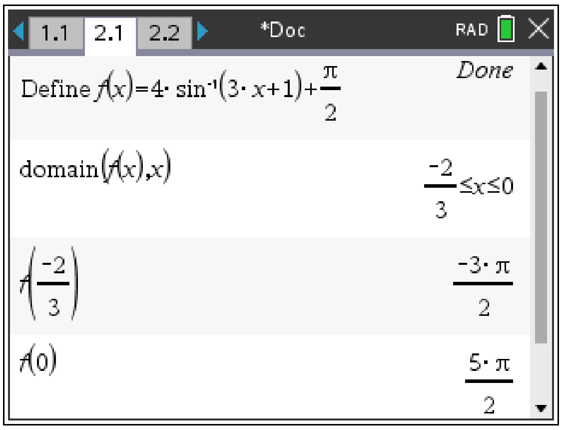
Answers

Question 1 Answer B



Question 2 Answer E



Note: When performing two or more operations with a function it is more efficient to define the function and then performing the required operations

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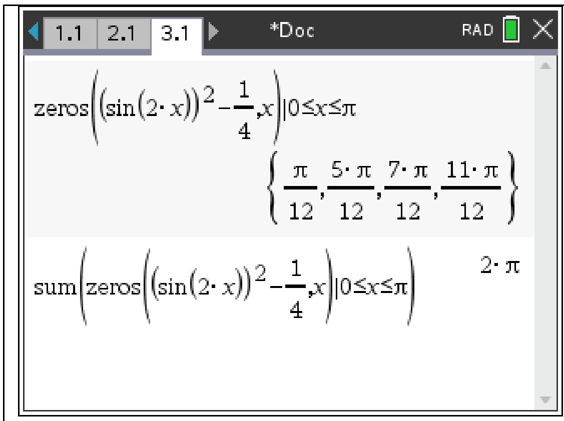
Question 3 **The sum of all the solutions is 2π**



To find the solutions we can use either the 'solve' command or the 'zeros' command.

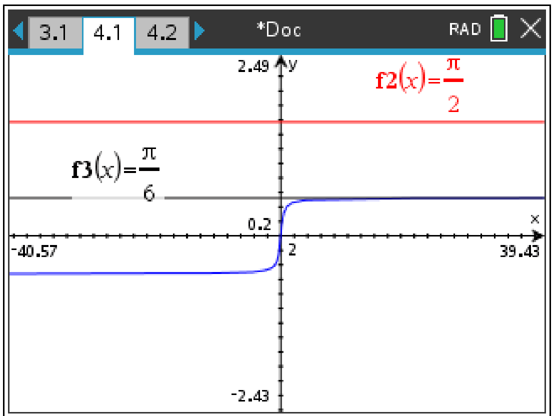
An equation $f(x) = g(x)$ can be written as $f(x) - g(x) = 0$.

To use 'zeros' type in zeros$(f(x) - g(x), x)$. It will return a list of the solutions to $f(x) = g(x)$. 'zeros' can also be obtained from the menu:

- Menu
- Algebra
- Zeros

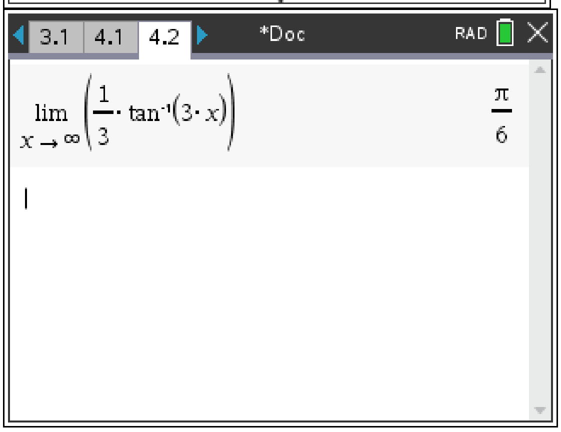
Once you obtain your list, you can type sum at the beginning to sum all of the elements in the list.

Question 4 **Answer C**



Note: The asymptotes are horizontal, so they must be of the form $y = a$, where a is a real number.

It can be seen graphically that the asymptotes occur at $y = \pm\frac{\pi}{6}$ as the arctan graph approaches the line $y = \frac{\pi}{6}$.



Alternatively, you can use the limit template to obtain the asymptote.

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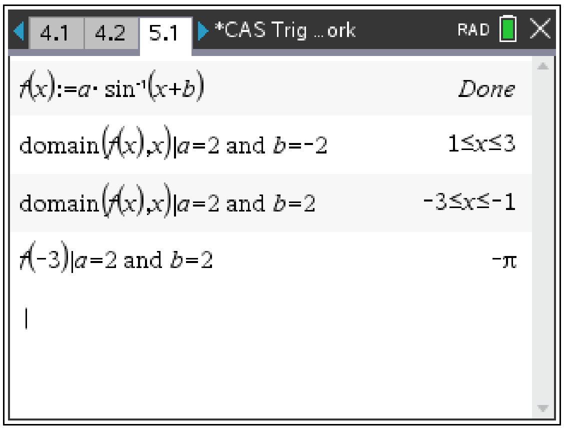
Question 5 Answer B

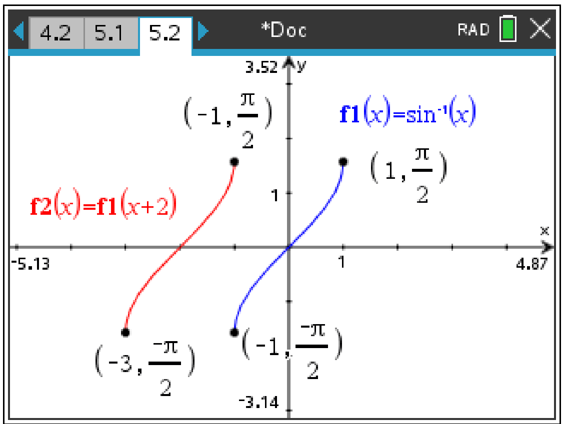
Method 1: Trial and Error

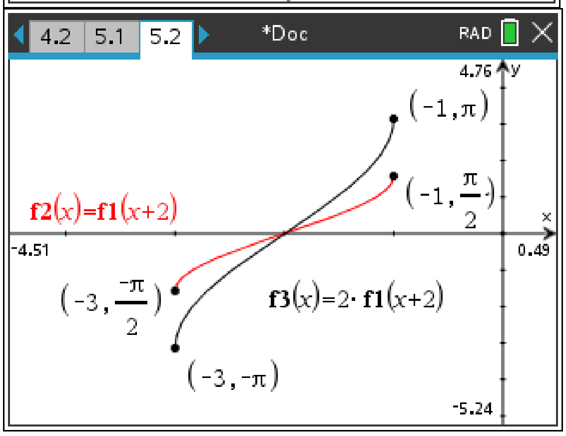


Values for a and b can be substituted by using the 'given' or 'such that' command, |

Once the correct domain is identified, the range can be determined by substituting in the endpoints.

Method 2: Horizontal shift, then dilation





Note: By sketching $f1(x) = sin^{-1}(x)$, you can apply transformations to $y = f1(x)$ to manipulate the endpoints to fit the required domain and range.

Method 3: Dilation, then horizontal shift

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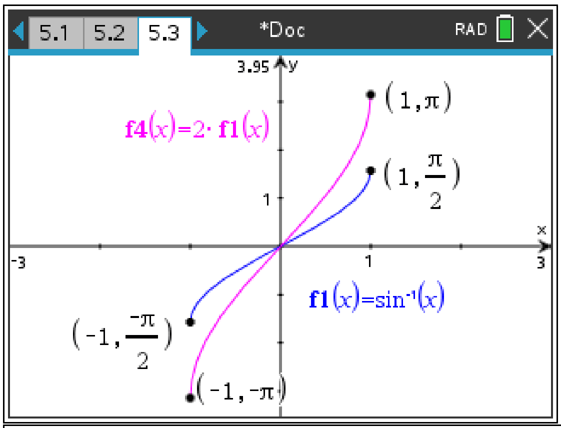
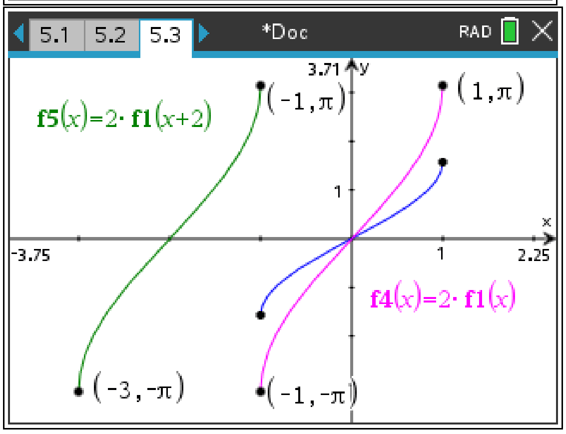
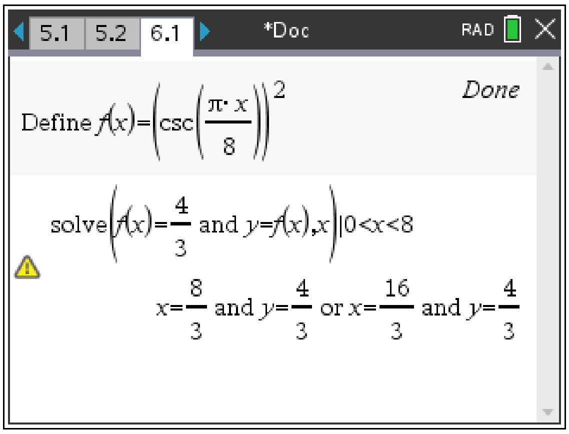




Question 7 $\left(\frac{8}{3},\frac{4}{3}\right)$ and $\left(\frac{16}{3},\frac{4}{3}\right)$



Note: When defining $cosec^2\left(\frac{\pi x}{8}\right)$ on CAS, you must type it in as $\left(cosec\left(\frac{\pi x}{8}\right)\right)^2$. This is the same for any trigonometric function.

Note: When solving trigonometric function, do not forget to put a domain restriction after the solve command, otherwise you will obtain general solutions.

Note: A nice way to obtain 'coordinates' is to simultaneously solve for x and to also solve for $y = f(x)$, where $f(x)$ is your defined function. The CAS will output the corresponding x and y values.

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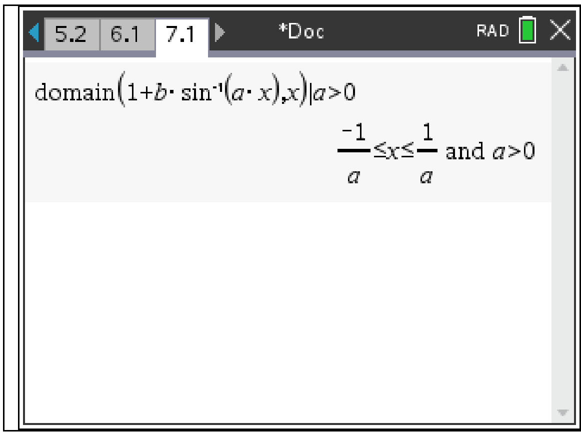
Question 8 Answer C

	**Note: The CAS Calculator can work with literal equations and expressions. It is important to specify which is the required variable in the equation or expression.**

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