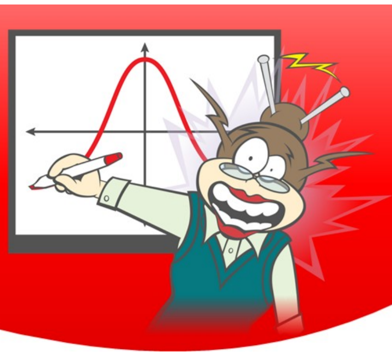
STUDENT REVISION SERIES



Graph Sketching Part 1

Question: 1.

Given that $f(x)=\begin{cases}\left|x^2-1\right|, & x\geq 0\\ 1-x^3, & x<0\end{cases}$, it follows that $f'\left(\frac{1}{2}\right)$ equals:

A. 0 B. 1 C. -1 D. $-\frac{3}{4}$ E. $\frac{3}{4}$

Question: 2.

Consider $h(x)=\arccos\left(|x|\right)$.

The number of solutions to the equation $h(x)=a,\ 0\leq a<\frac{\pi}{2}$, is

A. 4 B. 1 C. 0 D. 2 E. 3

Question: 3.

Which of the following statements is false, given that $f(x)=\log_e\left(\left|x+\sqrt{x^2-4}\right|\right)$?

A. $f(-2)=f(2)$

B. $f'(x)$ is undefined for $-2<x<2$

C. $f(x)$ is concave up for $x<2$

D. $f'(4)=\frac{\sqrt{3}}{6}$

E. $f(-3)=f(3)$

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Question: 4.

Let

$$g(x)=\begin{cases}\arcsin(x) & -1\le x<\dfrac{1}{\sqrt{2}}\\ ax^2+bx, & x\ge\dfrac{1}{\sqrt{2}}\end{cases}$$

a) Find the values of a and b so that g is continuous and differentiable over its domain.

b) Sketch the graphs of $g(x)$ and $g'(x)$ using the obtained values of a and b.

Question: 5.

Consider the pair of graphs $y=|x|$ and $y=-|x|+b$, where $b\in\mathbb{R}^+$.
If the graphs enclose an area of 20 square units, find the value of b.

Question: 6.

For any function, $g(x)$, continuous and differentiable over its domain, the following is always true:

A. $g(|x|)\ge 0$

B. the graphs of $|g(x)|$ and $g(|x|)$ are symmetrical about the y-axis

C. $|g(x)|'$ is a continuous function

D. $g(|x|)$ is an even function

E. $|g(x)|$ is an odd function

Question: 7.

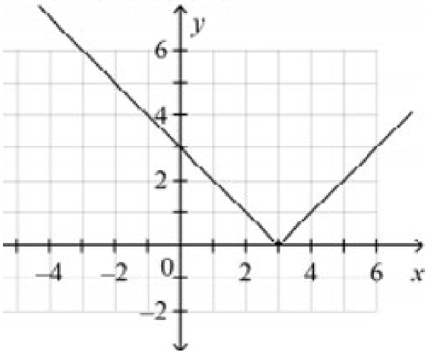
The functions f and g are defined as

$$f(x)=2|x|+3$$
$$g(x)=3-4x$$

Solve $f(g(x))>f(x)$.

Question: 8.

The equation of the following function is $y = |ax + b|,\ a < 0$.



The value of b is:

A. -3

B. -1

C. 1

D. 3

E. $-\frac{1}{3}$

Question: 9.

Let $f(x) = |x|$. The graph of f is transformed by:

- a dilation by a factor of 3 from the x-axis, followed by
- a translation of 1 unit horizontally to the right, followed by
- a dilation by a factor of $\frac{1}{2}$ from the y-axis.

The rule of the transformed graph is

A. $h(x) = 2|3x + 1|$

B. $h(x) = 3|2x - 1|$

C. $h(x) = 3|2(x - 1)|$

D. $h(x) = 2\left|\frac{x}{3} - 1\right|$

E. $h(x) = 3\left|\frac{x - 1}{2}\right|$

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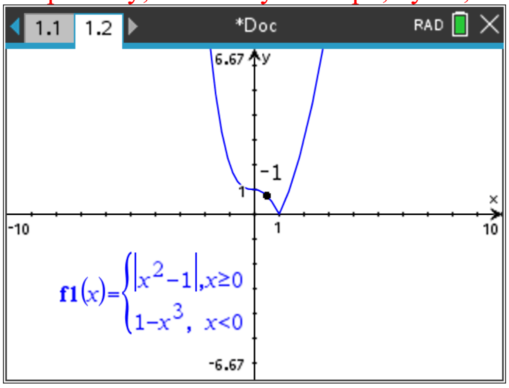
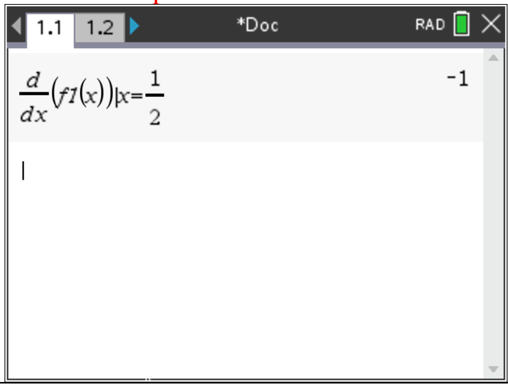
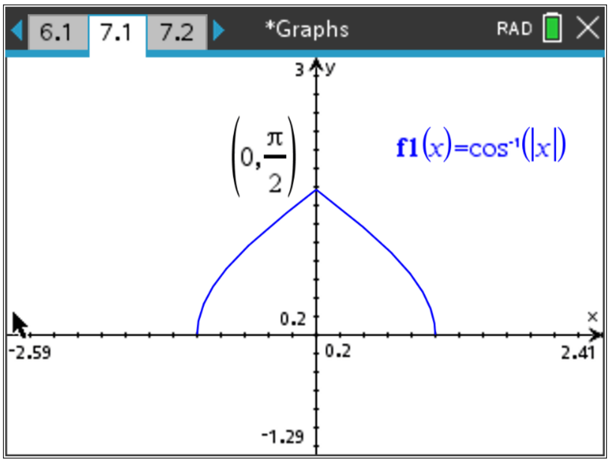
Answers

Question 1 Answer: C

Graphically, use Analyze Graph, dy/dx, enter ½.	Derivative at a point once the function is defined as a piecewise function.
	

Question 2 Answer: D

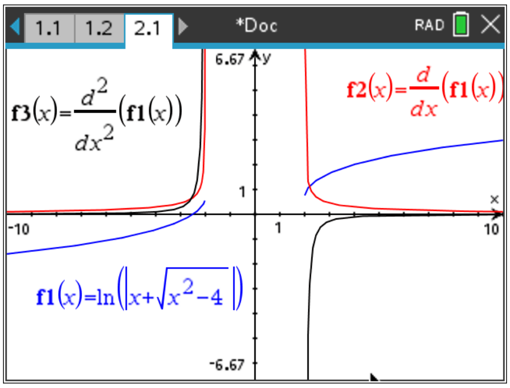
There will be two solutions as can be seen from the graph below:



The horizontal line $y = a,\ 0 \leq a < \frac{\pi}{2}$, will cut twice the given graph.

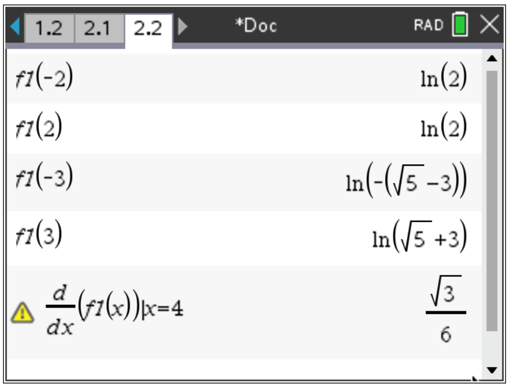
Question 3 Answer: E

Draw $f(x)$, $f'(x)$ and $f''(x)$ on one set of axes:



From the diagram B and C are correct.

Check the values

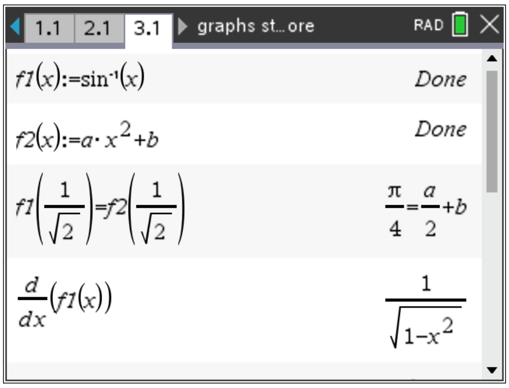


A and D are correct

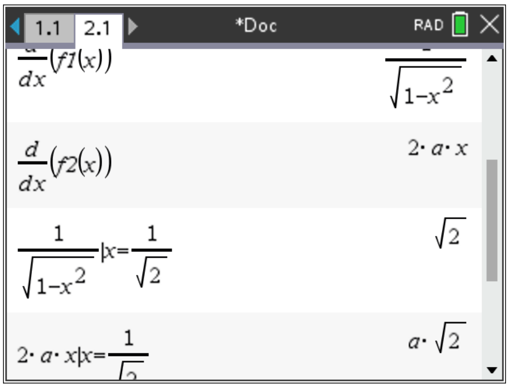
Option E is incorrect

Question 4

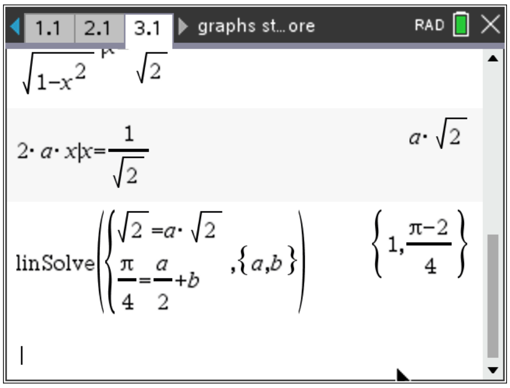
a) $a=1,\ b=\dfrac{\pi-2}{4}$



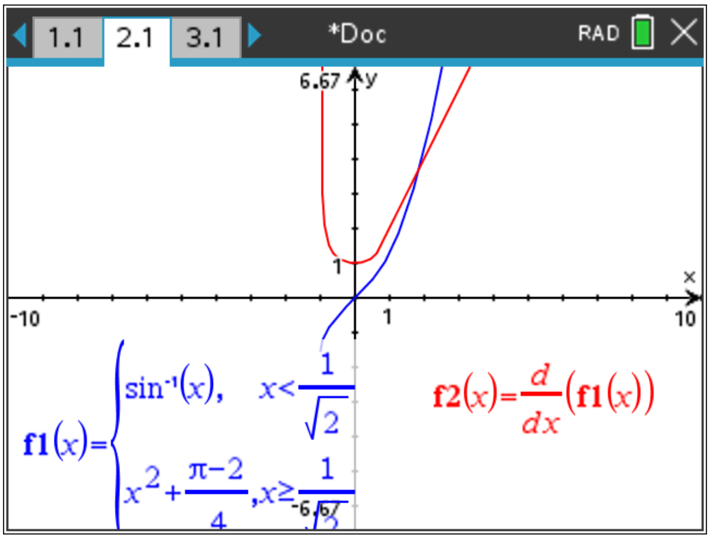
Define both functions separately and equate at the point of contact.



Equate derivatives at the point of contact and solve simultaneously for *a* and *b*.

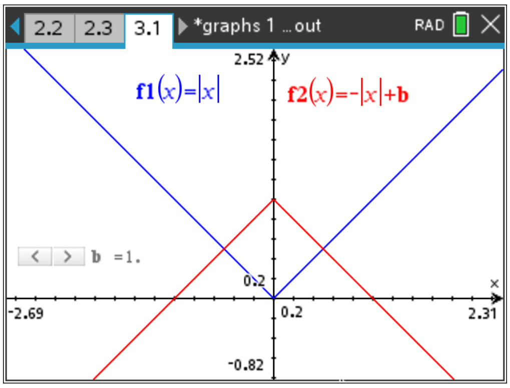


b)

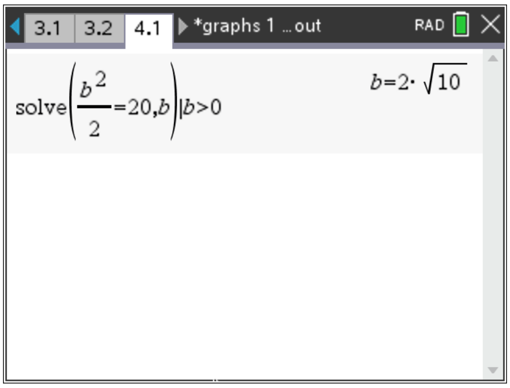


Question 5

$b = 2\sqrt{10}$



It can be seen that the shape is a square with diagonal length equal to b



Square side has length a such that:

$$a^2 + a^2 = b^2$$

$$2a^2 = b^2$$

$$a^2 = \frac{b^2}{2}$$

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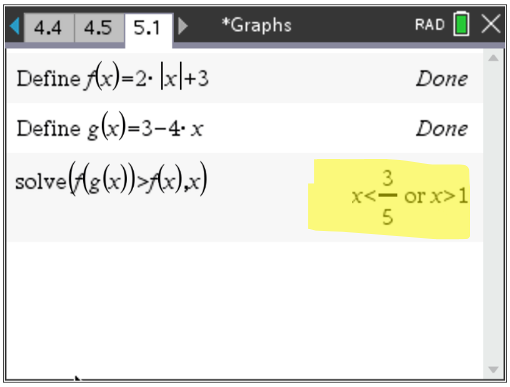
Question 6 Answer: D

$g(|x|)$ is always an even function, as $g(-x)=g(x)$ and the graph is symmetrical about the y-axis.

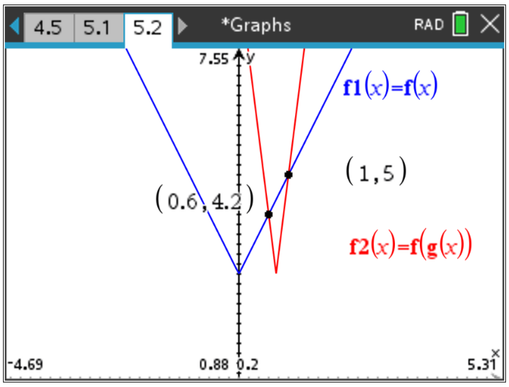
Question 7

$x<\dfrac{3}{5}\ or\ x>1.$

Define and solve

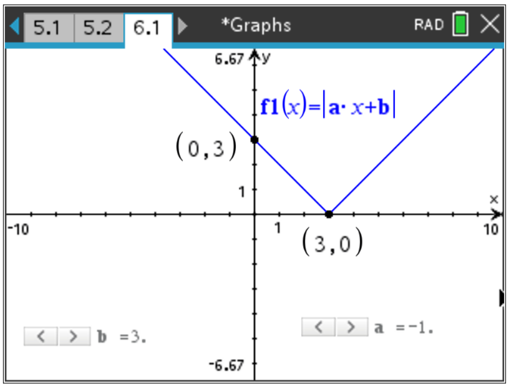


Graphically



Question 8 Answer: B

Using sliders



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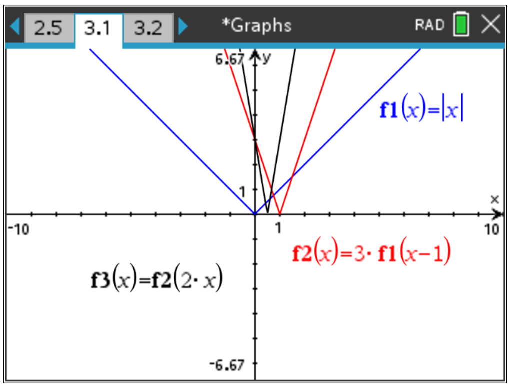
Question 9 Answer: B

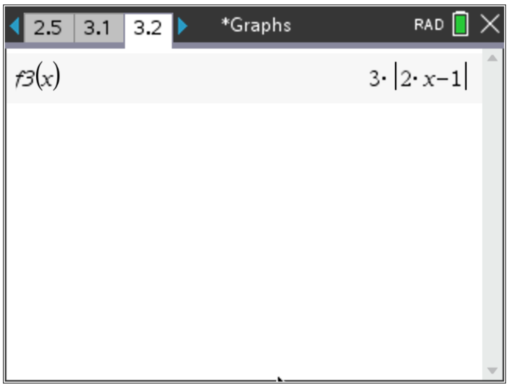
Define $f(x)=\|x\|$	*Done*
$3\cdot f(x-1)$	$3\cdot \|x-1\|$
$g(x):=3\cdot \|x-1\|$	*Done*
$g(2\cdot x)$	$3\cdot \|2\cdot x-1\|$

Alternatively draw it:

Follow the order of transformations



Find the rule for the final graph



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