# STUDENT REVISION SERIES

## **Mathematical Methods**



Unit 2 Topic 3 Trigonometric Graphs

Each of the questions included here can be solved using either the TI-Nspire CX or CX CAS.

#### **Question 1**

Find all solutions to the equation  $\cos(x) = \frac{1}{\sqrt{2}}$  for  $x \in [0, 4\pi]$ .

#### **Question 2**

Find all solutions to the equation  $2\sin(x) - \sqrt{3} = 0$  for  $x \in [0, 2\pi]$ 

#### **Question 3**

Sketch the graph of  $y = 2 \sin \left(x - \frac{\pi}{2}\right) + 1$  for  $x \in [0, 2\pi]$ . List the x intercepts of the graph for this interval.

#### **Question 4**

Sketch the graph of  $y = 2\cos\left(x + \frac{\pi}{6}\right) - 1$  for  $x \in [0, 2\pi]$ . List the *x* intercepts of the graph for this interval.

#### **Question 5**

The temperature, T°C, in a town over a day is modelled by the function with the rule  $T = 17 - 8\cos\left(\frac{\pi t}{12}\right)$ 

© Texas Instruments 2020. You may copy, communicate and modify this material for non-commercial educational purposes provided all acknowledgements associated with this material are maintained.



where *t* is the time in hours after midnight,  $0 \le t \le 24$ 

a) What is the temperature at (i) 3am (ii) 2pm?

b) What are the minimum and maximum temperatures for the domain specified?

c) At what times of the day, to the nearest minute, are temperatures warmer than 20°C?

d) Sketch the graph for the temperatures over a day.

### Answers

Question 1 © Texas Instruments 2020. You may copy, communicate and modify this material for non-commercial educational purposes provided all acknowledgements associated with this material are maintained.

Using a geometry command in a Graph application: Menu > Geometry > Points and Lines > Intersection point(s) Choose the required graphs and all intersection points are labelled



Using Numerical Solve in a Calculator application; divide by  $\pi$ ; Menu > Number > Approximate to fraction. To find subsequent solutions: copy the initial nSolve equation down, and add in the restriction x > previous answer

1.1 1.2 *	Doc RAD	×
nSolve $\left(\cos(x)=\frac{1}{\sqrt{2}},x\right)$	0.785398	
0.78539816339744 π	0.25	l
0.25▶approxFraction(5	$(E-14)$ $\frac{1}{4}$	
1.1 1.2	Doc RAD	1
n Solve $\left(\cos(x) = \frac{1}{\sqrt{2}}, x\right)$	Doc RAD	
n Solve $\left(\cos(x) = \frac{1}{\sqrt{2}}, x\right)$	Doc RAD	
$\frac{1.1}{n \text{ Solve}} \left( \cos(x) = \frac{1}{\sqrt{2}}, x \right)   x$ $\frac{7.068583470577}{\pi}$	Doc RAD 55.4977871437821 7.06858 2.25	
1.1 1.2 nSolve $\left(\cos(x) = \frac{1}{\sqrt{2}}, x\right)   x$ $\frac{7.068583470577}{\pi}$ 2.25 approx Fraction (5	Doc RAD 5.4977871437821 7.06858 2.25 .E-14) <u>9</u> 4	

1.1 1.2 ▶ *Doc	RAD 📘 🗡
nSolve $\left(\cos(x)=\frac{1}{\sqrt{2}},x\right) x>0.7853$	9816339744
	5.49779
5.4977871437821 π	1.75
1.75▶approxFraction(5.E-14)	<u>7</u> 4
< 1.1 1.2 ▶ *Doc	RAD 🗐 🗡
1.1 1.2 *Doc nSolve $\left(\cos(x) = \frac{1}{\sqrt{2}}, x\right)$  x>7.0685	RAD () > 583470577 11.781
■ 1.1 1.2 ■ *Doc nSolve( $cos(x) = \frac{1}{\sqrt{2}}, x$ ) x>7.0685 <u>11.780972450962</u> π	RAD >
↓ 1.1 1.2 → *Doc nSolve $\left(\cos(x) = \frac{1}{\sqrt{2}}, x\right)$  x>7.0685 11.780972450962 $\frac{11.780972450962}{\pi}$ 3.750000000000   1 → approx Fract	RAD >

#### Question 2

:

Find exact solutions and verify with graph.



1.2 2.1 2.2 ▶ *Doc	rad 📋 🗙
nSolve $\left(\sin(x) = \frac{\sqrt{3}}{2}, x\right)  x>1.0$	471975511966
11 C	2.0944
2.0943951023932	0.666667
π	
0.66666666666667 approx	Fraction(5.E-14)
	2
	3 🗸





#### Question 3

Sketch the graph in a Graph application. Use the Window settings to make the *x* scale equal to *C* (phase shift).





Consider the tick marks on the x axis, to refine the scale. In this case, visually a better scale is  $\frac{\pi}{6}$ . Change the scale to read off the x intercepts.





© Texas Instruments 2020. You may copy, communicate and modify this material for non-commercial educational purposes provided all acknowledgements associated with this material are maintained.

#### Question 4

Sketch the graph in a Graph application. Use the Window settings to make the x scale equal to C (phase shift).





Consider the tick marks on the x axis, and read off the x intercepts Change the scale if required to read off the x intercepts.

: the x intercepts are 
$$\frac{\pi}{6}$$
;  $\frac{9\pi}{6}$  which simplifies to  $\frac{\pi}{6}$ ;  $\frac{3\pi}{2}$ 

#### **Question 5**

i)

Sketch the graph in a Graph application. Set the window to reflect the domain and adjust the range to view the graph.



2.2 3 V	Window	v Settings	-BAD 🗋 🗙
1	×Min:	0	$\frac{\pi}{\epsilon}$ -1
	XMax:	2π	
0:5	, XScale:	π/6	
0.01024	YMin:	-5	0.20
	YMax:	3	(x) = -1
	YScale:	Auto 🕨	7
		OK Cance	- 1
-5	-	(200) 2009-0	

a) Place a point on the graph using the Geometry tools: Menu > Geometry > Points & Lines > Point On Remember click once to select the graph, again to place the point and then ESC to drop the tool. Double click on the x coordinate and type the required x value. Repeat to find the y value when x = 14

			<i>,</i>	
🗲 1 Actions 🛛 🕨	rad 📋 🗙		< 3.1 3.2 4.1 ▶	*Doc RAD 📕 🗙
			30 V	
<ul> <li>1 Point</li> </ul>	(p)			(14, 22, 0)
🕶 2 Point On				(14,23.9)
(,) 3 Point by Coordinates	(p) $f_2(x) = 17$			$f_2(x) = 17$
ightarrow 4 Intersection Point(s)				
🛹 5 Line				
- 6 Segment	ints & Lines 🔸	Place	(3,11.3)	
<ul> <li>7 Ray</li> </ul>	apes 🕨			$f1(x)=17-8 \cdot \cos\left(\frac{\pi \cdot x}{x}\right)$
🕂 8 Tangent	asurement 🤸		2	(12) ×
🛹 9 Vector	nstruction		0 1	24
🔿 A Circle arc	ansformation 🕨		-6.67	
The temperature a	t 3am is 11.3°C	ii)	The tempera	ture at 2pm is 23.9°C

b) Use the Analyse Graph tool to find the maximum and minimum of the function.

<b>\$</b> 1 /	Actions	🕨 c 🖬 🗎 🗙	3.1 3.2	4.1	▶ *Doc	RAD 📋 🕽
😼 2 🕚	View	•	30 Y		(10.05)	
Ab 3 (	Graph Entry/Edit	(14,23.9)			(12,25)(	14,23.9)
Q4 V	Window / Zoom			/		
1 5 -	Trace	2ero 1 Zero	1	/		$f_2(x)=17$
66	Analyze Granh	Sin 2 Minimum		/		/
图 7 -	Table	Max 3 Maximum				(24.0
		$\times$ 4 Intersection	(3,1	1.3)		(-24,9
•≥ 8 (	Geometry	- 5 dv/dx		10		$(\pi \cdot x)$
<b>Q</b> 9 5	Settings	▶ 6 Integral	2	1	$II(x) = 17 - 8 \cdot 6$	cos 12
01	• • • • • • • • •	🕷 7 Bounded Area	0 1	• • •	• • • • • • •	· · · · · · · · · · · · · · · · · · ·
-6.67		⊙ 8 Analyze Conics ▸	-6.67			

The minimum temperature is  $9^{\circ}C$  and the maximum is  $25^{\circ}C$ 

c) This can also be done using the graph, but using Numerical Solve in a Calculator application allows for calculating to the nearest minute more easily.

Convert the decimal part of the solution to minutes to generate the answers of 7: 28 and 14:32



3.2 4.1 4.2 ▶ *Doc	rad 📘 🗙
nSolve $(fI(x)=20,x)$	7.46829
7.468287522469-7	0.468288
0.468287522469.60	28.0973
E	

The temperature is warmer than 20°C at 7: 28 and 14:32

d)





© Texas Instruments 2020. You may copy, communicate and modify this material for non-commercial educational purposes provided all acknowledgements associated with this material are maintained.

