

# Solving differential equations worksheet

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Each of the questions included here can be solved using the TI-Nspire CX CAS.

## Question 1

Find the possible values of  $a$  and  $b$  such that  $x = t(a \cos(2t) + b \sin(2t))$  is the solution to the differential equation

$$\frac{d^2x}{dt^2} + 4x = 2 \cos(2t).$$

Response:

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## Question 2

Find the equation of the curve given by the following equation and which passes through the given point:

$$\frac{dy}{dx} = 2x^2 e^{-y}: \quad x = 0, \quad y = 0$$

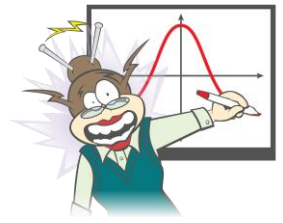
Response:

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### Question 3

The differential equation satisfied by  $y = e^{kx^2}$  is:

**A**  $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = 0$

**B**  $\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 6y = 0$

**C**  $\frac{d^2 y}{dx^2} - 2kx \frac{dy}{dx} + 2ky = 0$

**D**  $\frac{d^2 y}{dx^2} - 2kx \frac{dy}{dx} - 2ky = 0$

**E**  $\frac{d^2 y}{dx^2} - 2kx \frac{dy}{dx} - y = 0$

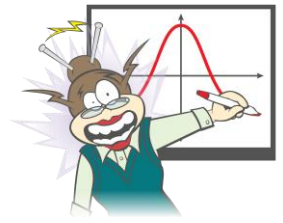
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**Question 4**

Solve  $\frac{dx}{dt} = \sqrt{1-x^2}$ ;  $x\left(-\frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

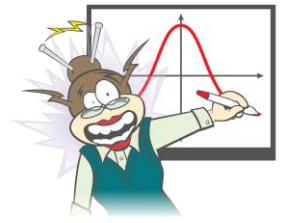
Response:

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## Answers

### Question 1

Answer:  $b = \frac{1}{2}$  and  $a = 0$

A screenshot of a TI-84 Plus calculator interface. The top of the screen shows '1.1' and '2.1' in the window and 'RAD' in the mode. The main display shows the equation  $x(t) := t \cdot (a \cdot \cos(2 \cdot t) + b \cdot \sin(2 \cdot t))$  with 'Done' to the right. Below this, the first derivative is shown as  $\frac{d}{dt}(x(t)) = (2 \cdot b \cdot t + a) \cdot \cos(2 \cdot t) + (b - 2 \cdot a \cdot t) \cdot \sin(2 \cdot t)$ . The second derivative is shown as  $\frac{d^2}{dt^2}(x(t)) = (4 \cdot b - 4 \cdot a \cdot t) \cdot \cos(2 \cdot t) + (-4 \cdot b \cdot t - 4 \cdot a) \cdot \sin(2 \cdot t)$ .

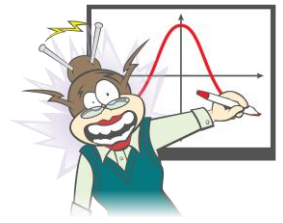
$$\frac{d^2}{dt^2}(x(t)) + 4 \cdot x(t) = 2 \cdot \cos(2 \cdot t)$$

$$4 \cdot b \cdot \cos(2 \cdot t) - 4 \cdot a \cdot \sin(2 \cdot t) = 2 \cdot \cos(2 \cdot t)$$

Equating coefficients on both sides of the equation gives:

$$4b = 2 \Rightarrow b = \frac{1}{2}$$

$$-4a = 0 \Rightarrow a = 0$$



## Question 2

Answer:  $y = \ln \left| \frac{2x^3}{3} + 1 \right|$

$$\frac{dy}{dx} = 2x^2 e^{-y} : x=0, y=0$$

$$\int e^y dy = \int 2x^2 dx$$

$$e^y = \frac{2x^3}{3} + c$$

$$\Rightarrow e^y = \frac{2x^3}{3} + 1$$

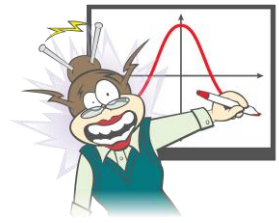
$$y = \ln \left| \frac{2x^3}{3} + 1 \right|$$

## Question 3

Answer: D

Substituting to D gives:

$$\begin{aligned} & (4k^2x^2 + 2k)e^{kx^2} - 2kx(2kxe^{kx^2}) - 2ke^{kx^2} \\ &= (4k^2x^2 + 2k - 4k^2x^2 - 2k)e^{kx^2} \\ &= 0 \end{aligned}$$



#### Question 4

Answer:  $x = \sin\left(t + \frac{7\pi}{12}\right)$

$$\int \frac{dx}{\sqrt{1-x^2}} = \int dt$$

$$\sin^{-1}(x) = t + c$$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{3} + c \Rightarrow c = \frac{7\pi}{12}$$

$$\sin^{-1}(x) = t + \frac{7\pi}{12}$$

$$x = \sin\left(t + \frac{7\pi}{12}\right)$$

The screenshot shows a TI-84 Plus calculator interface with the following content:

- Top bar: 1.1 2.1 \*Doc RAD
- Input:  $\text{deSolve}(x'=\sqrt{1-x^2} \text{ and } x\left(\frac{-\pi}{3}\right)=\frac{1}{\sqrt{2}},t,x)$
- Output:  $\sin^{-1}(x) - \frac{\pi}{4} = t + \frac{\pi}{3}$
- Output:  $\left(\sin^{-1}(x) - \frac{\pi}{4} = t + \frac{\pi}{3}\right) + \frac{\pi}{4}$
- Output:  $\sin^{-1}(x) = t + \frac{7\pi}{12}$