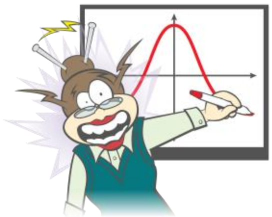
Solving differential equations worksheet



Author: Bozenna Graham

Each of the questions included here can be solved using the TI-Nspire CX CAS.

Question 1

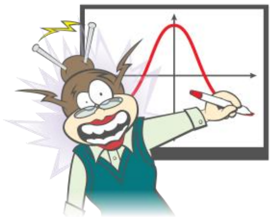
Find the possible values of a and b such that $x = t\,(a\cos(2t) + b\sin(2t))$ is the solution to the differential equation $\frac{d^2x}{dt^2} + 4x = 2\cos(2t)$.

Response:

Question 2

Find the equation of the curve given by the following equation and which passes through the given point:
$\frac{dy}{dx} = 2x^2e^{-y}$: $x = 0,\ y = 0$

Response:



Question 3

The differential equation satisfied by $y = e^{kx^2}$ is:

A $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} = 0$

B $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 6y = 0$

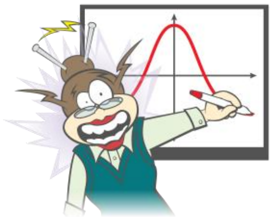
C $\dfrac{d^2y}{dx^2} - 2kx\dfrac{dy}{dx} + 2ky = 0$

D $\dfrac{d^2y}{dx^2} - 2kx\dfrac{dy}{dx} - 2ky = 0$

E $\dfrac{d^2y}{dx^2} - 2kx\dfrac{dy}{dx} - y = 0$

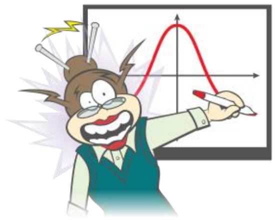
Response:

Question 4



Solve $\frac{dx}{dt} = \sqrt{1-x^2};\ x\left(-\frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

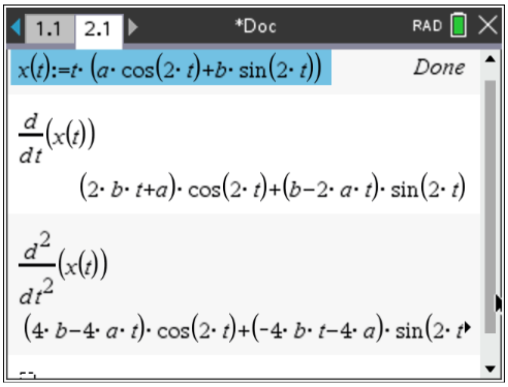
Response:



Answers

Question 1

Answer: $b = \frac{1}{2}$ and $a = 0$



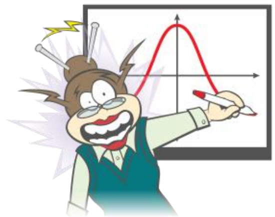
$$\frac{d^2}{dt^2}(x(t))+4\cdot x(t)=2\cdot \cos(2\cdot t)$$

$$4\cdot b\cdot \cos(2\cdot t)-4\cdot a\cdot \sin(2\cdot t)=2\cdot \cos(2\cdot t)$$

Equating coefficients on both sides of the equation gives:

$$4b = 2 \Rightarrow b = \frac{1}{2}$$

$$-4a = 0 \Rightarrow a = 0$$



Question 2

Answer: $y=\ln\left|\frac{2x^3}{3}+1\right|$

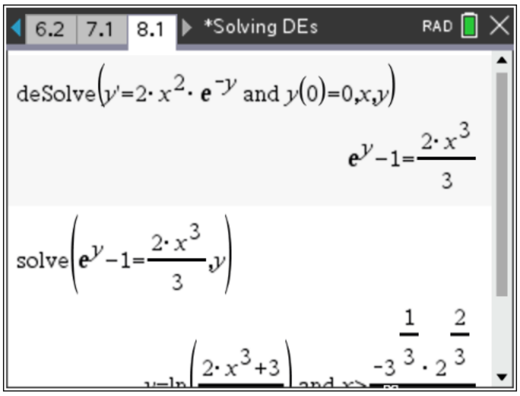
$$\frac{dy}{dx}=2x^2e^{-y}:\ \ x=0,\ y=0$$

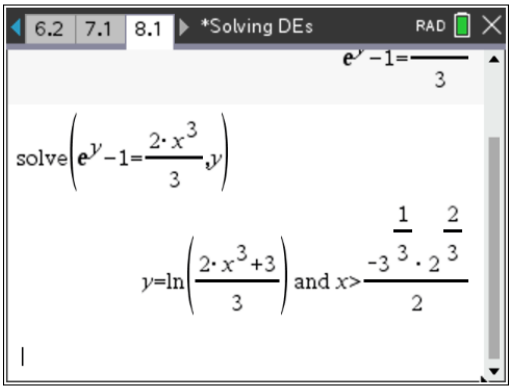
$$\int e^y\ dy=\int 2x^2 dx$$

$$e^y=\frac{2x^3}{3}+c$$

$$\Rightarrow e^y=\frac{2x^3}{3}+1$$

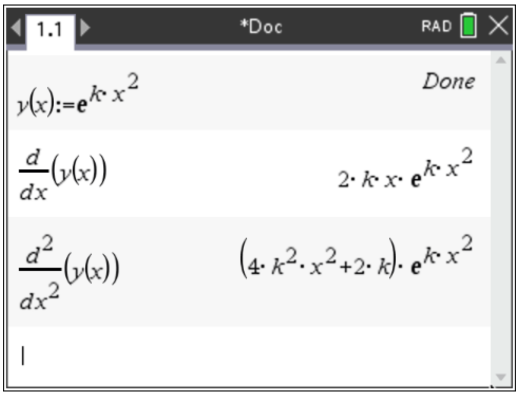
$$y=\ln\left|\frac{2x^3}{3}+1\right|$$





Question 3

Answer: **D**

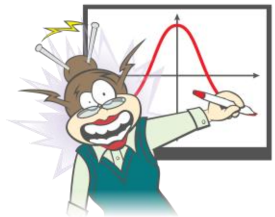


Substituting to D gives:

$$\left(4k^2x^2+2k\right)e^{kx^2}-2kx\left(2kxe^{kx^2}\right)-2ke^{kx^2}$$

$$=\left(4k^2x^2+2k-4k^2x^2-2k\right)e^{kx^2}$$

$$=0$$



Question 4

Answer: $x=\sin\left(t+\frac{7\pi}{12}\right)$

$$\int\frac{dx}{\sqrt{1-x^2}}=\int dt$$

$$\sin^{-1}(x)=t+c$$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=-\frac{\pi}{3}+c\Rightarrow c=\frac{7\pi}{12}$$

$$\sin^{-1}(x)=t+\frac{7\pi}{12}$$

$$x=\sin\left(t+\frac{7\pi}{12}\right)$$